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An Equilateral Triangle Centered about the Centroid of a Triangle

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1. Introduction

We provide justification for the claim made in Euclid #8414.

2. Results

Theorem 2.1. *Let ABC be a triangle with P a point in the plane.*
Let $A'B'C'$ be the first Napoleon triangle.
Let A_b be the reflection of A about line PB'.
Define A_c, B_a, B_c, C_a, and C_b cyclically.
Let P_A be the circumcenter of AB_cC_b.
Define P_B and P_C cyclically.
Then the centroids of AP_BP_C, BP_AP_C, and CP_AP_B form an equilateral triangle centered at the centroid of ABC.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
P has coordinates (x_P, y_P).
A' has coordinates
$((x_C+1+x_C/2+(3^{1/2}*y_C)/2+1/2)/3, (y_C+(y_C-3^{1/2}*(x_C-1))/2)/3) = (x_D, y_D)$.
B' has coordinates
$((x_C + x_C/2 - (3^{1/2} * y_C)/2)/3, (y_C + y_C/2 + (3^{1/2} * x_C)/2)/3) = (x_E, y_E)$.
C' has coordinates

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$(1/2, -3^{1/2}/6) = (x_F, y_F)$.
Line PA' is given by the equation $y = y_P + (x - x_P) * (y_P - y_D)/(x_P - x_D)$.
Line PB' is given by the equation $y = y_P + (x - x_P) * (y_P - y_E)/(x_P - x_E)$.
Line PC' is given by the equation $y = y_P + (x - x_P) * (y_P - y_F)/(x_P - x_F)$.
A_b has coordinates
$(2 * (((y_P - y_E) * (x_E * y_P - x_P * y_E))/((y_P - y_E)^2 + (x_P - x_E)^2)), 2 * (-((x_P - x_E) * (x_E * y_P - x_P * y_E))/((y_P - y_E)^2 + (x_P - x_E)^2)))$
$= (x_G, y_G)$.
A_c has coordinates
$(2 * (((y_P - y_F) * (x_F * y_P - x_P * y_F))/((y_P - y_F)^2 + (x_P - x_F)^2)), 2 * (-((x_P - x_F) * (x_F * y_P - x_P * y_F))/((y_P - y_F)^2 + (x_P - x_F)^2)))$
$= (x_H, y_H)$.
B_a has coordinates
$(2 * ((x_D * y_P^2 - x_P * y_D * y_P - x_D * y_D * y_P + x_P * y_D^2 + x_P^2 - 2 * x_D * x_P + x_D^2)/((y_P - y_D)^2 + (x_P - x_D)^2)) - 1, 2 * (-((x_P - x_D) * ((x_D - 1) * y_P - (x_P - 1) * y_D))/((y_P - y_D)^2 + (x_P - x_D)^2)))$
$= (x_I, y_I)$.
B_c has coordinates
$(2 * ((x_F * y_P^2 - x_P * y_F * y_P - x_F * y_F * y_P + x_P * y_F^2 + x_P^2 - 2 * x_F * x_P + x_F^2)/((y_P - y_F)^2 + (x_P - x_F)^2)) - 1, 2 * (-((x_P - x_F) * ((x_F - 1) * y_P - (x_P - 1) * y_F))/((y_P - y_F)^2 + (x_P - x_F)^2)))$
$= (x_J, y_J)$.
C_a has coordinates
$(2 * ((x_D * y_P^2 - x_P * y_D * y_P - x_D * y_D * y_P + x_P * y_C * y_P - x_D * y_C * y_P + x_P * y_D^2 - x_P * y_C * y_D + x_D * y_C * y_D + x_C * x_P^2 - 2 * x_C * x_D * x_P + x_C * x_D^2)/((y_P - y_D)^2 + (x_P - x_D)^2)) - x_C, 2 * ((y_C * y_P^2 - (2 * y_C * y_D + (x_D - x_C) * (x_P - x_D)) * y_P + y_C * y_D^2 + (x_P - x_C) * (x_P - x_D) * y_D)/((y_P - y_D)^2 + (x_P - x_D)^2)) - y_C)$
$= (x_K, y_K)$.
C_b has coordinates
$(2 * ((x_E * y_P^2 - x_P * y_E * y_P - x_E * y_E * y_P + x_P * y_C * y_P - x_E * y_C * y_P + x_P * y_E^2 - x_P * y_C * y_E + x_E * y_C * y_E + x_C * x_P^2 - 2 * x_C * x_E * x_P + x_C * x_E^2)/((y_P - y_E)^2 + (x_P - x_E)^2)) - x_C, 2 * ((y_C * y_P^2 - (2 * y_C * y_E + (x_E - x_C) * (x_P - x_E)) * y_P + y_C * y_E^2 + (x_P - x_C) * (x_P - x_E) * y_E)/((y_P - y_E)^2 + (x_P - x_E)^2)) - y_C)$
$= (x_L, y_L)$.
P_A has coordinates
$(-(y_J * y_L^2 - (y_J^2 + x_J^2) * y_L + x_L^2 * y_J)/(2 * (x_J * y_L - x_L * y_J)), (x_J * y_L^2 - x_L * y_J^2 + x_J * x_L^2 - x_J^2 * x_L)/(2 * (x_J * y_L - x_L * y_J)))$
$= ((4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)), -(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-$

$3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))).$

P_B has coordinates

$(-(y_H*y_K^2+(-y_H^2-x_H^2+1)*y_K+(x_K^2-1)*y_H)/(2*((x_H-1)*y_K+(1-x_K)*y_H)),((x_H-1)*y_K^2+(1-x_K)*y_H^2+(x_H-1)*x_K^2+(1-x_H^2)*x_K+x_H^2-x_H)/(2*((x_H-1)*y_K+(1-x_K)*y_H)))$

$=(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2),-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)).$

P_C has coordinates

$(-((y_G-y_C)*y_I^2+(-y_G^2+y_C^2-x_G^2+x_C^2)*y_I+y_C*y_G^2+(-y_C^2+x_I^2-x_C^2)*y_G+(x_G^2-x_I^2)*y_C)/(2*((x_G-x_C)*y_I+(x_C-x_I)*y_G+(x_I-x_G)*y_C)),((x_G-x_C)*y_I^2+(x_C-x_I)*y_G^2+(x_I-x_G)*y_C^2+(x_G-x_C)*x_I^2+(x_C^2-x_G^2)*x_I+x_C*x_G^2-x_C^2*x_G)/(2*((x_G-x_C)*y_I+(x_C-x_I)*y_G+(x_I-x_G)*y_C)))$

$=(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2),-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)).$

The centroid of ABC has coordinates $((x_C+1)/3,y_C/3)$.

The centroid of AP_BP_C has coordinates

$((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3,(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*$

$y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3).$

The centroid of BP_AP_C has coordinates

$((1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3, (-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3).$

The centroid of CP_AP_B has coordinates

$((x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*$

$y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2))))/3, (y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3).$
We verify that the centroids of AP_BP_C, BP_AP_C, and CP_AP_B are centered about the centroid of ABC since we calculate
$((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-$

$x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3)/3 = (x_C+1)/3$

and

$((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3)/3 = y_C/3.$

Now, to show that the centroids of AP_BP_C, BP_AP_C, and CP_AP_B form an equilateral triangle, it suffices to show that their orthocenter is also the centroid of

ABC.
The altitude of this triangle through the centroid of AP_BP_C is given by the equation
$y = (-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(x-((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))*(-1)/(((-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*$

$3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))/((1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-(x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3)).$

The altitude of this triangle through the centroid of BP_AP_C is given by the equation

$y=(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+(x-((1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*$

$$y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))*(-1)/(((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))/(((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*$$

$(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))).$

We verify that the centroid of ABC lies on both of these lines since we calculate

$(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+((x_C+1)/3-((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))*(-1)/(((-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*$

$x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))/((1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-(x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))-y_C/3=0,$

and

$(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*$

$$y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3+((x_C+1)/3-((1+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))*(-1)/(((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(27*x_P+9*3^{1/2}*y_P+54*x_C*x_P+18*y_C*y_P-9*x_C*(x_C+2)-54*x_P^2+9*y_C^2-18*3^{1/2}*x_C^2*y_P-54*3^{1/2}*x_P^2*y_C-18*3^{1/2}*y_C*y_P^2+18*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_C*y_C+36*3^{1/2}*x_P*y_C-18*3^{1/2}*x_P*y_P+36*3^{1/2}*x_C*x_P*y_C+36*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((y_C+(-(((2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2+(2*x_P-1)*(3*x_P+3^{1/2}*y_P-1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+(2*x_P-1)^2*(3*y_P-3^{1/2}*x_P+3^{1/2})^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+(2*x_P-1)^2*(3*x_P+3^{1/2}*y_P-1)^2*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*y_C-18*3^{1/2}*y_P-36*y_C*y_P+9*x_C*(2*x_C-3*x_P*(x_C-2*x_P+2)+1)-27*x_P*y_C^2+18*y_C^2+9*3^{1/2}*x_C^2*y_P-18*3^{1/2}*y_C*y_P^2+9*3^{1/2}*y_C^2*y_P-18*3^{1/2}*x_P*y_C+36*3^{1/2}*x_P*y_P+54*x_P*y_C*y_P-18*3^{1/2}*x_C*x_P*y_P))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3))/((-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)+(-((y_C*(18*x_C-36*x_P-9*3^{1/2}*y_C*(2*x_P-1)+18)-y_P*(18*3^{1/2}*(y_C-2*x_C*y_C+3*x_C*y_P-2*x_P*y_C)-54*x_P+27)+9*3^{1/2}*(x_P*((2*x_C+2)*(x_C-x_P)+1)-x_C^2))*(3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2))))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3-((x_C+(4*(2*x_P-1)^2*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2)+3^{1/2}*$$

$(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(x_C-3*x_P-3^{1/2}*y_C+3^{1/2}*y_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)+3^{1/2}*(2*x_P-1)*(3^{1/2}*y_P-x_P+1)*(y_C-3*y_P+3^{1/2}*x_C-3^{1/2}*x_P)^2*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)^2*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1))/(8*(2*x_P-1)*(-x_C^2+2*x_P*x_C-y_C^2+2*y_P*y_C)*(3*x_P^2-3*x_P+3*y_P^2+3^{1/2}*y_P+1)*(y_C-3^{1/2}*x_C+2*3^{1/2}*x_P-3*3^{1/2}*x_P^2-3*3^{1/2}*y_P^2+2*3^{1/2}*x_C*x_P+2*3^{1/2}*y_C*y_P)*(x_C^2-3*x_C*x_P-3^{1/2}*x_C*y_P+3*x_P^2+3^{1/2}*x_P*y_C+y_C^2-3*y_C*y_P+3*y_P^2))+(-((3*3^{1/2}*y_P^2-2*3^{1/2}*y_C*y_P+y_C+3^{1/2}*(x_C-x_P*(2*x_C-3*x_P+2)))*(9*3^{1/2}*x_C^2*x_P+27*x_C^2*y_P-18*3^{1/2}*x_C*x_P^2-54*x_C*x_P*y_P+18*3^{1/2}*x_C*x_P-9*3^{1/2}*x_C-18*3^{1/2}*x_P^2+9*3^{1/2}*x_P*y_C^2-18*3^{1/2}*x_P*y_C*y_P+18*x_P*y_C+18*3^{1/2}*x_P+27*y_C^2*y_P-54*y_C*y_P^2+36*3^{1/2}*y_C*y_P+9*y_C-54*3^{1/2}*y_P^2))/(54*(x_P*(2*x_C-3*x_P+2)-x_C+y_P*(2*y_C-3*y_P))^2-18*y_C^2)))/3)))-y_C/3=0$
as desired. □

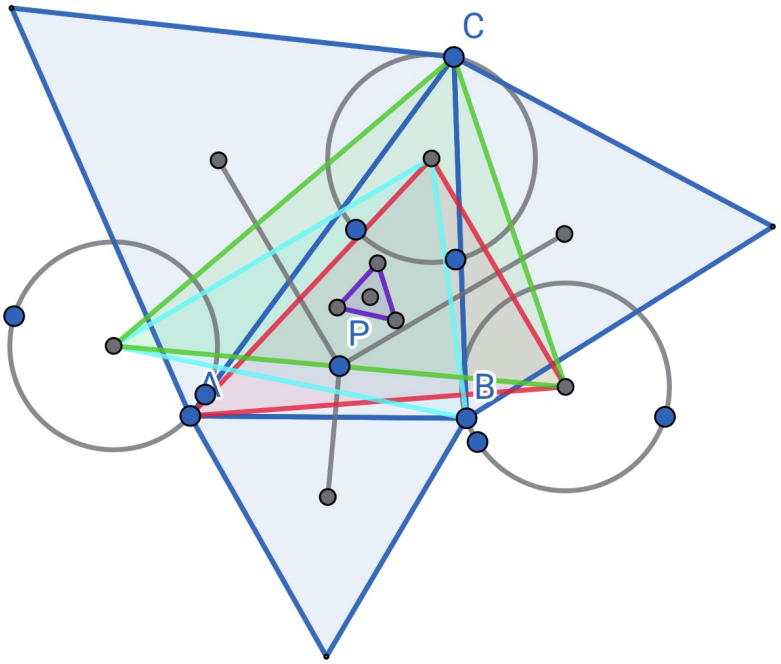

Figure 1. Illustration.

References

[1] Weisstein, Eric W. "Napoleon's Theorem." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/NapoleonsTheorem.html