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Two Pedal Triangles and a Circle

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1. Introduction

We provide a proof of the statement proclaimed in Euclid #8372.

2. Results

Theorem 2.1. *Let ABC be a triangle and P, Q points in the plane with $A'B'C'$ and $A''B''C''$ the pedal triangles of P and Q respectively.*
Let $A_1B_1C_2$ be the reflection triangle of P about $A''B''C''$.
Let $A_2B_2C_2$ be the reflection triangle of $A_1B_1C_1$ about $A'B'C'$.
Then the circumcircle of $A_2B_2C_2$ is identical to the circle whose radius is the chord connecting P and the reflection of Q about P.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
A' has coordinates
$((y_C * ((x_C - 1) * y_P + y_C) + (x_C - 1)^2 * x_P)/(y_C^2 + (x_C - 1)^2), (y_C * (y_C * y_P + (x_C - 1) * x_P - x_C + 1))/(y_C^2 + (x_C - 1)^2))$.
B' has coordinates
$((x_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2), (y_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2))$.
C' has coordinates $(x_P, 0)$.
A'' has coordinates
$((y_C * ((x_C - 1) * y_Q + y_C) + (x_C - 1)^2 * x_Q)/(y_C^2 + (x_C - 1)^2), (y_C * (y_C * y_Q +$

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$(x_C - 1) * x_Q - x_C + 1))/(y_C^2 + (x_C - 1)^2))$.
B'' has coordinates
$((x_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2), (y_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2))$.
C'' has coordinates $(x_Q, 0)$.
A_1 has coordinates
$(2 * ((y_C * ((x_C - 1) * y_Q + y_C) + (x_C - 1)^2 * x_Q)/(y_C^2 + (x_C - 1)^2)) - x_P, 2 * ((y_C * (y_C * y_Q + (x_C - 1) * x_Q - x_C + 1))/(y_C^2 + (x_C - 1)^2)) - y_P)$.
B_1 has coordinates
$(2 * ((x_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2)) - x_P, 2 * ((y_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2)) - y_P)$.
C_1 has coordinates
$(2 * x_Q - x_P, -y_P)$.
A_2 has coordinates
$(2 * ((y_C * ((x_C - 1) * y_P + y_C) + (x_C - 1)^2 * x_P)/(y_C^2 + (x_C - 1)^2)) - (2 * ((y_C * ((x_C - 1) * y_Q + y_C) + (x_C - 1)^2 * x_Q)/(y_C^2 + (x_C - 1)^2)) - x_P), 2 * ((y_C * (y_C * y_P + (x_C - 1) * x_P - x_C + 1))/(y_C^2 + (x_C - 1)^2)) - (2 * ((y_C * (y_C * y_Q + (x_C - 1) * x_Q - x_C + 1))/(y_C^2 + (x_C - 1)^2)) - y_P))$.
B_2 has coordinates
$(2 * ((x_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2)) - (2 * ((x_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2)) - x_P), 2 * ((y_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2)) - (2 * ((y_C * (x_C * x_Q + y_C * y_Q))/(x_C^2 + y_C^2)) - y_P))$.
C_2 has coordinates
$(2 * x_P - (2 * x_Q - x_P), y_P)$.
Consider the circle centered at the reflection of Q about P, which passes through P, whose equation is given by
$(x-(2*x_P-x_Q))^2+(y-(2*y_P-y_Q))^2-(x_P-(2*x_P-x_Q))^2-(y_P-(2*y_P-y_Q))^2 = 0$.
We verify that A_2, B_2, and C_2 lie on this circle since we calculate
$(2 * ((y_C * ((x_C - 1) * y_P + y_C) + (x_C - 1)^2 * x_P)/(y_C^2 + (x_C - 1)^2)) - (2 * ((y_C * ((x_C - 1) * y_Q + y_C) + (x_C - 1)^2 * x_Q)/(y_C^2 + (x_C - 1)^2)) - x_P) - (2 * x_P - x_Q))^2 + (2 * ((y_C * (y_C * y_P + (x_C - 1) * x_P - x_C + 1))/(y_C^2 + (x_C - 1)^2)) - (2 * ((y_C * (y_C * y_Q + (x_C - 1) * x_Q - x_C + 1))/(y_C^2 + (x_C - 1)^2)) - y_P) - (2 * y_P - y_Q))^2 - (x_P - (2 * x_P - x_Q))^2 - (y_P - (2 * y_P - y_Q))^2 = 0$,
$(2*((x_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-(2*((x_C*(x_C*x_Q+y_C*y_Q))/(x_C^2+y_C^2))-x_P)-(2*x_P-x_Q))^2+(2*((y_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-(2*((y_C*(x_C*x_Q+y_C*y_Q))/(x_C^2+y_C^2))-y_P)-(2*y_P-y_Q))^2-(x_P-(2*x_P-x_Q))^2-(y_P-(2*y_P-y_Q))^2 = 0$,
and
$(2 * x_P - (2 * x_Q - x_P) - (2 * x_P - x_Q))^2 + (y_P - (2 * y_P - y_Q))^2 - (x_P - (2 * x_P - x_Q))^2 - (y_P - (2 * y_P - y_Q))^2 = 0$.

□

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[1] Weisstein, Eric W. "Pedal Triangle." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/PedalTriangle.html

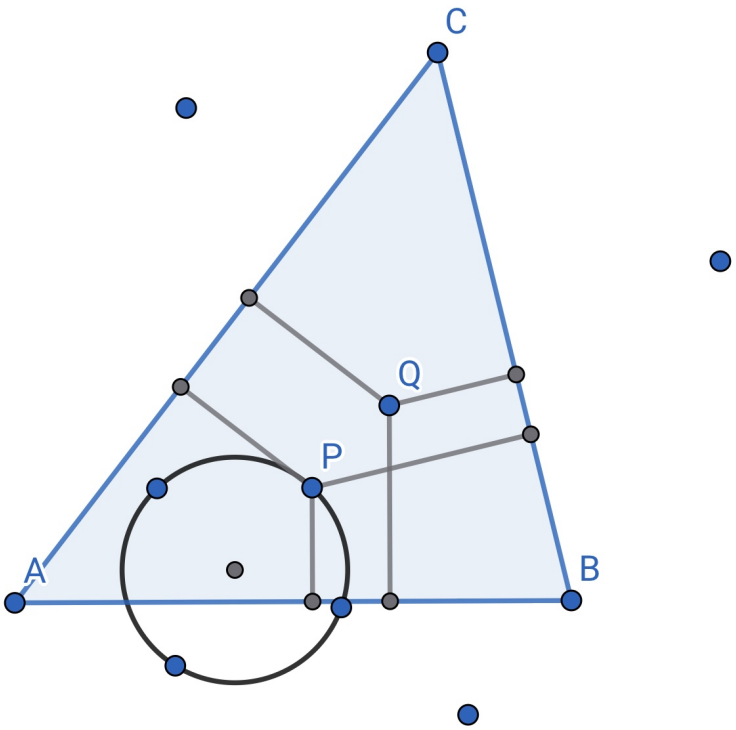

Figure 1. Illustration.