International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 10, 2025, pp. 193-194.
Received March 29, 2025. Published on-line July 16, 2025
web: http://www.journal-1.eu/
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# Tangents and a Parabola

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**Mathematics Subject Classification (2010).** 51N20.

## 1. Introduction

We prove a result about a parabola whose focus and directrix are a vertex and opposite side of a given triangle.

## 2. Results

**Theorem 2.1.** *Let $ABC$ be a triangle.*
*Let $w$ be the parabola whose focus is $A$ and whose directrix is line $BC$.*
*Let $w_b$ and $w_c$ be the points on $w$ whose tangent lines are parallel to lines $AC$ and $AB$ respectively.*
*Let $P$ be the midpoint of $w_b$ and $w_c$.*
*Let the lines tangent to $w$ at $w_b$ and $w_c$ intersect at $Q$.*
*Then the midpoint of $P$ and $Q$ is the point on $w$ whose tangent line is parallel to line $w_bw_c$.*

*Proof.* Without loss of generality, we will work in Cartesian coordinates.
$A$ has coordinates $(x_A, y_A)$.
$B$ has coordinates $(0, 0)$.
$C$ has coordinates $(1, 0)$.
$w$ is given by the equation $y = ((x - x_A)^2 + y_A^2)/(2 * y_A)$.
$w$ has derivative $\frac{dy}{dx} = (x - x_A)/y_A$.
Line $AB$ has slope $y_A/x_A$.
Line $AC$ has slope $y_A/(x_A - 1)$.
$w_b$ has coordinates $(x_A + y_A^2/x_A, y_A^3/(2 * x_A^2) + y_A/2)$.
$w_c$ has coordinates $(y_A^2/(x_A - 1) + x_A, y_A^3/(2 * (x_A - 1)^2) + y_A/2)$.

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The line tangent to $w$ at $w_b$ is given by the equation
$y = (y_A * (x_A * (2 * x - x_A) - y_A^2))/(2 * x_A^2)$.
The line tangent to $w$ at $w_c$ is given by the equation
$y = (y_A * (2 * (x_A - 1) * x - y_A^2 - x_A^2 + 1))/(2 * (x_A - 1)^2)$.
$P$ has coordinates
$((2 * x_A + y_A^2/x_A + y_A^2/(x_A - 1))/2, (y_A^3/(2 * x_A^2) + y_A + y_A^3/(2 * (x_A - 1)^2))/2)$.
$Q$ has coordinates
$(((2 * x_A - 1) * y_A^2)/(2 * (x_A - 1) * x_A) + x_A, y_A^3/(2 * (x_A - 1) * x_A) + y_A/2)$.
Let $R$ be the midpoint of $P$ and $Q$, which has coordinates
$(((2*x_A-1)*y_A^2)/(2*(x_A-1)*x_A)+x_A, ((2*x_A-1)^2*y_A^3)/(8*(x_A-1)^2*x_A^2)+y_A/2)$.
We verify that $R$ lies on $w$ since we calculate $((x_R - x_A)^2 + y_A^2)/(2 * y_A) - y_R$
$= ((((2 * x_A - 1) * y_A^2)/(2 * (x_A - 1) * x_A) + x_A - x_A)^2 + y_A^2)/(2 * y_A) - (((2 * x_A - 1)^2 * y_A^3)/(8 * (x_A - 1)^2 * x_A^2) + y_A/2) = 0$.
We then verify that the tangent line at $R$ is parallel to line $w_b w_c$ since we calculate
$(y_{w_b} - y_{w_c})/(x_{w_b} - x_{w_c}) - \frac{dy}{dx}|_{x=x_R}$
$= ((y_A^3/(2 * x_A^2) + y_A/2) - (y_A^3/(2 * (x_A - 1)^2) + y_A/2))/((x_A + y_A^2/x_A) - (y_A^2/(x_A - 1) + x_A)) - (((2 * x_A - 1) * y_A^2)/(2 * (x_A - 1) * x_A) + x_A - x_A)/y_A = 0$ as desired. □

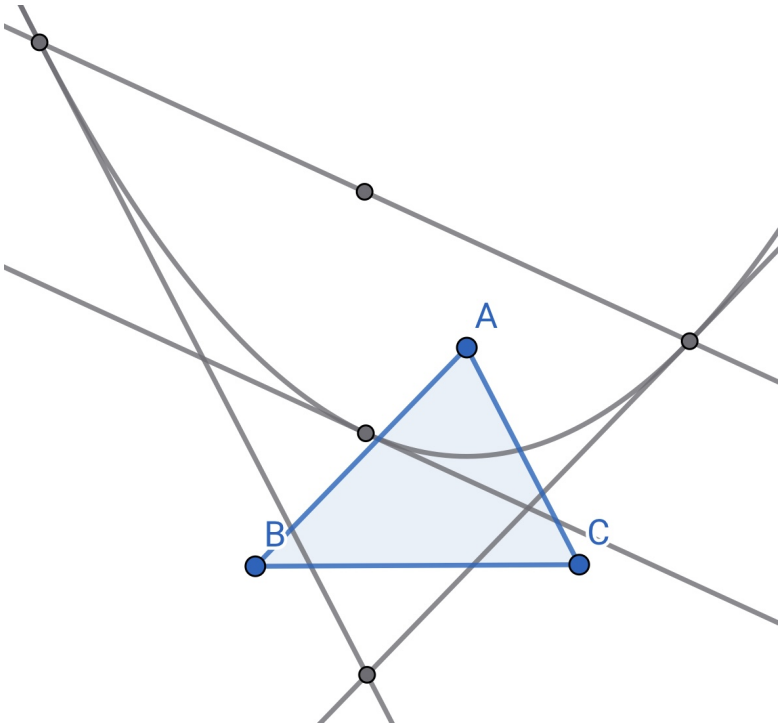


FIGURE 1. Illustration.

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