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Three Points on the Orthocentroidal Circle

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1. Introduction

The orthocentroidal circle of a triangle ABC is the circle whose diameter is the chord connecting the centroid and the orthocenter. We demonstrate here that three more points lie on this circle.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be the centroid of ABC.
Let E be the orthocenter of ABC.
Let F, G, and H be the midpoints of AB, AC, and BC.
Let I, J, and K be the vertices of the circumcevian triangle of D adjacent to sides AB, AC, and BC respectively.
Let I', J', and K' be the reflections of I, J, and K about F, G, and H respectively.
Then I', J', and K' lie on the orthocentroidal circle of ABC.

Proof. A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates $((x_C + 1)/3, y_C/3)$.
E has coordinates $(x_C, ((1 - x_C) * x_C)/y_C)$.
F has coordinates $(1/2, 0)$.
G has coordinates $(x_C/2, y_C/2)$.
H has coordinates $((x_C + 1)/2, y_C/2)$.
I has coordinates

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$((2*y_C^2+(x_C-1)*(2*x_C-1))/(4*y_C^2+(2*x_C-1)^2), -y_C/(4*y_C^2+(2*x_C-1)^2))$.
J has coordinates
$(((x_C-1)*(y_C^2+(x_C-2)*x_C))/(y_C^2+(x_C-2)^2), (y_C*(y_C^2+x_C^2-2*x_C+2))/(y_C^2+(x_C-2)^2))$.
K has coordinates
$(((x_C+1)*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2), (y_C*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2))$.
I' has coordinates
$(1-(2*y_C^2+(x_C-1)*(2*x_C-1))/(4*y_C^2+(2*x_C-1)^2), y_C/(4*y_C^2+(2*x_C-1)^2))$.
J' has coordinates
$(x_C-((x_C-1)*(y_C^2+(x_C-2)*x_C))/(y_C^2+(x_C-2)^2), y_C-(y_C*(y_C^2+x_C^2-2*x_C+2))/(y_C^2+(x_C-2)^2))$.
K' has coordinates
$(x_C+1-((x_C+1)*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2), y_C-(y_C*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2))$.
The orthocentroidal circle of ABC is given by the equation
$(x-(((x_C+1)/3+x_C)/2))^2+(y-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2-(((x_C+1)/3-(((x_C+1)/3+x_C)/2))^2+(y_C/3-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2)=0$.
We then verify that I', J', and K' lie on the orthocentroidal circle since we calculate
$(1-(2*y_C^2+(x_C-1)*(2*x_C-1))/(4*y_C^2+(2*x_C-1)^2)-(((x_C+1)/3+x_C)/2))^2+(y_C/(4*y_C^2+(2*x_C-1)^2)-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2-(((x_C+1)/3-(((x_C+1)/3+x_C)/2))^2+(y_C/3-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2)=0$,
$(x_C-((x_C-1)*(y_C^2+(x_C-2)*x_C))/(y_C^2+(x_C-2)^2)-(((x_C+1)/3+x_C)/2))^2+(y_C-(y_C*(y_C^2+x_C^2-2*x_C+2))/(y_C^2+(x_C-2)^2)-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2-(((x_C+1)/3-(((x_C+1)/3+x_C)/2))^2+(y_C/3-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2)=0$,
and
$(x_C+1-((x_C+1)*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2)-(((x_C+1)/3+x_C)/2))^2+(y_C-(y_C*(y_C^2+x_C^2+1))/(y_C^2+(x_C+1)^2)-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2-(((x_C+1)/3-(((x_C+1)/3+x_C)/2))^2+(y_C/3-(y_C/3+((1-x_C)*x_C)/y_C)/2)^2)=0$ as desired. □

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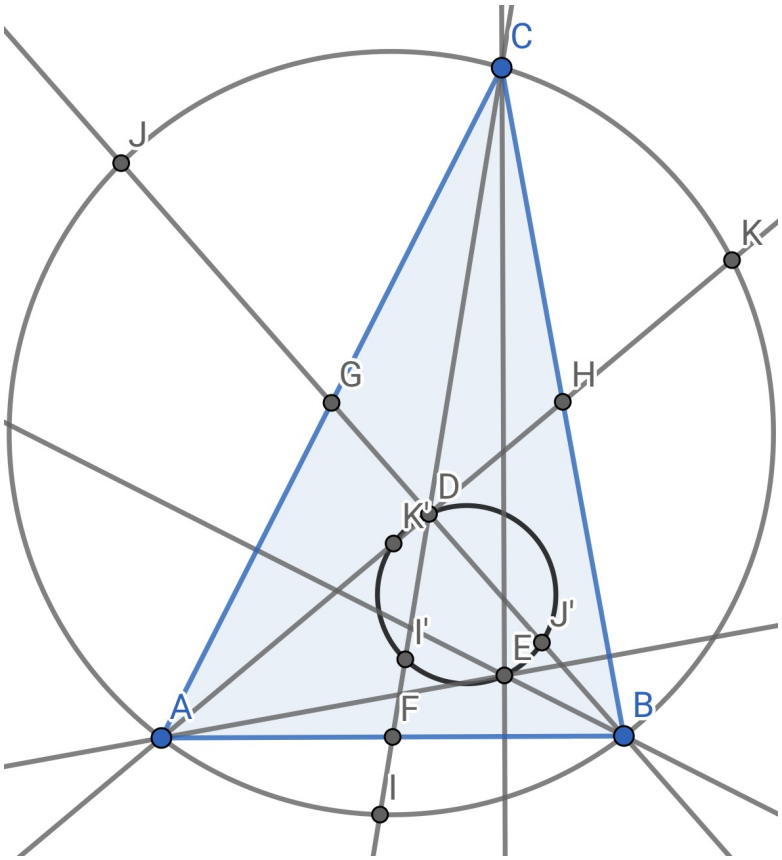

Figure 1. Illustration.