International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 9, 2024, pp. 454-456.
Received October 8, 2024. Published on-line December 31, 2024.
web: http://www.journal-1.eu/
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Simple Construction of a Cleaver

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Mathematics Subject Classification (2010). 51N20.

1. Introduction

Pick a midpoint between two of the vertices of a given triangle. The line which passes through that midpoint and which bisects the perimeter of the triangle is known as a cleaver. Here, we will provide a short way to construct a cleaver.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle, where $BC < AC$.*
Let F be the point on line BC, where $\angle ACF = \pi - \angle ACB$, such that $CF = AC$.
Let G be the point on line AC, where $\angle BCG = \pi - \angle ACB$, such that $CG = BC$.
Let H be the midpoint of A and B.
Let I be the midpoint of F and G.
Let lines HI and AC intersect at J.
Then the line segment HJ is a cleaver of ABC.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C) where $x_C > 1/2$.
F has coordinates.
$(x_C + ((x_C^2 + y_C^2)^{1/2} * (x_C - 1))/((x_C - 1)^2 + y_C^2)^{1/2}, (y_C * (x_C + ((x_C^2 + y_C^2)^{1/2} *$

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$(x_C-1))/((x_C-1)^2+y_C^2)^{1/2}-1))/(x_C-1))$.
G has coordinates.
$((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(x_C^2+y_C^2),((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(x_C^2+y_C^2))*(y_C/x_C))$.
H has coordinates $(1/2,0)$.
I has coordinates $((x_C+((x_C^2+y_C^2)^{1/2}*(x_C-1))/((x_C-1)^2+y_C^2)^{1/2}+(x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(x_C^2+y_C^2))/2,((y_C*(x_C+((x_C^2+y_C^2)^{1/2}*(x_C-1))/((x_C-1)^2+y_C^2)^{1/2}-1))/(x_C-1)+((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(x_C^2+y_C^2))*(y_C/x_C))/2)$.
Line HI is given by the equation
$y=((2*x-1)*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}-x_C*(x_C-1)+y_C^2))/(y_C*(4*x_C-2))$.
J has coordinates
$((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2),((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2))*(y_C/x_C))$.
We calculate that
$BC+CJ-AJ=\sqrt{(x_C-1)^2+y_C^2}+\sqrt{(x_J-x_C)^2+(y_J-y_C)^2}-\sqrt{x_J^2+y_J^2}$
$=((x_C-1)^2+y_C^2)^{1/2}+(((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2)-x_C)^2+(((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2))*(y_C/x_C)-y_C)^2)^{1/2}-(((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2))^2+(((x_C*((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2}+x_C^2+y_C^2))/(2*x_C^2+2*y_C^2))*(y_C/x_C))^2)^{1/2}$
$=\frac{|(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-x_C^2-y_C^2|+(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-x_C^2-y_C^2}{2\sqrt{x_C^2+y_C^2}}$.
Since $x_C>1/2$, we have that $AC>BC \iff 0>\sqrt{(x_C-1)^2+y_C^2}-\sqrt{x_C^2+y_C^2}$
$\iff 0>(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-(\sqrt{x_C^2+y_C^2})^2$
$=(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-x_C^2-y_C^2$
which means that
$\frac{|(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-x_C^2-y_C^2|+(\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})-x_C^2-y_C^2}{2\sqrt{x_C^2+y_C^2}}=0$
so the line segment HJ is indeed a cleaver.

□

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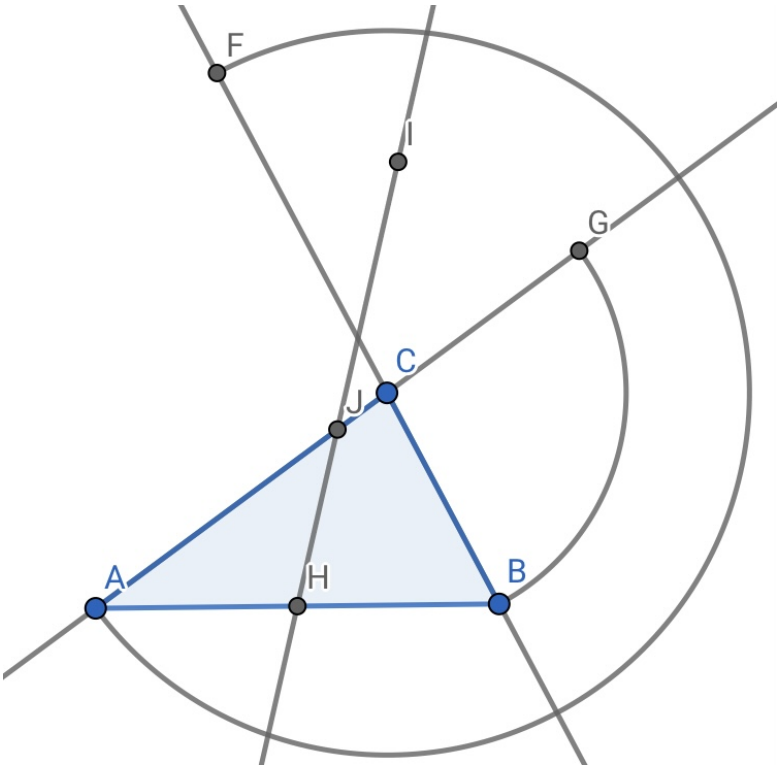

Figure 1. $BC + CJ = AJ$.