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A Third and Fourth Circle Passing Through the Bottema Point

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1. Introduction

In [1] and [2], we demonstrated the existence of several nontrivial circles which pass through the Bottema point. Here, we will demonstrate the existence of two more nontrivial circles passing through the Bottema point.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let $ACFG$ form a rectangle, with F adjacent to A.
Let $BCHI$ form a rectangle similar to $ACFG$, with I adjacent to B.
Let J be the Bottema point (the midpoint of F and I).
Let circles $ACFG$ and $BCHI$ intersect at K.
Then,
1. A, B, J, and K are concyclic, and
2. G, H, J, and K are concyclic.

Proof. A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
F has coordinates $(x_F, -(x_C * x_F)/y_C)$.
G has coordinates $(x_C + x_F, y_C - (x_C * x_F)/y_C)$.
H has coordinates $(x_C - x_F, y_C - ((1 - x_C) * x_F)/y_C)$.

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I has coordinates $(1 - x_F, -((1 - x_C) * x_F)/y_C)$.
J has coordinates $(1/2, -x_F/(2 * y_C))$.
K has coordinates
$(((x_F - x_C) * ((2 * x_F - 1) * y_C^2 - (2 * x_C - 1) * x_F^2))/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2), -(2 * x_F * (x_F - x_C) * (x_F + x_C - 1) * y_C)/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2))$.
Consider the circle whose equation is given by
$(x-1/2)^2+(y-(y_C/(4*x_F)-x_F/(4*y_C)))^2-1/4-(y_C/(4*x_F)-x_F/(4*y_C))^2 = 0$.
We verify that A, B, J, and K lie on this circle since we calculate that
$(0-1/2)^2+(0-(y_C/(4*x_F)-x_F/(4*y_C)))^2-1/4-(y_C/(4*x_F)-x_F/(4*y_C))^2 = 0$,
$(1-1/2)^2+(0-(y_C/(4*x_F)-x_F/(4*y_C)))^2-1/4-(y_C/(4*x_F)-x_F/(4*y_C))^2 = 0$,
$(1/2 - 1/2)^2 + (-x_F/(2 * y_C) - (y_C/(4 * x_F) - x_F/(4 * y_C)))^2 - 1/4 - (y_C/(4 * x_F) - x_F/(4 * y_C))^2 = 0$,
and
$(((x_F - x_C) * ((2 * x_F - 1) * y_C^2 - (2 * x_C - 1) * x_F^2))/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2) - 1/2)^2 + (-(2 * x_F * (x_F - x_C) * (x_F + x_C - 1) * y_C)/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2) - (y_C/(4 * x_F) - x_F/(4 * y_C)))^2 - 1/4 - (y_C/(4 * x_F) - x_F/(4 * y_C))^2 = 0$.
Next, consider the circle whose equation is given by
$(x - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (y - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2 - ((1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2) = 0$.
We verify that G, H, J, and K lie on this circle since we calculate that
$(x_C + x_F - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (y_C - (x_C * x_F)/y_C - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2 - ((1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2) = 0$,
$(x_C - x_F - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (y_C - ((1 - x_C) * x_F)/y_C - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2 - ((1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2) = 0$,
$(1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2 - ((1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2) = 0$,
and
$(((x_F - x_C) * ((2 * x_F - 1) * y_C^2 - (2 * x_C - 1) * x_F^2))/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2) - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-(2 * x_F * (x_F - x_C) * (x_F + x_C - 1) * y_C)/((2 * x_F - 1)^2 * y_C^2 + (2 * x_C - 1)^2 * x_F^2) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2 - ((1/2 - (((2 * x_C - 1) * x_F^2)/(4 * y_C^2) + (2 * x_C + 1)/4))^2 + (-x_F/(2 * y_C) - (y_C^2 + (x_F - 1) * x_F)/(2 * y_C))^2) = 0$ as desired. □

References

[1] Warren, B.L., "The Bottema Point Lies on a Circle", *International Journal of Computer Discovered Mathematics*, Under Review.

[2] Warren, B.L., "A Circle Passing Through the Bottema Point and a Circle Centered at the Bottema Point", *International Journal of Computer Discovered Mathematics*, Under Review.

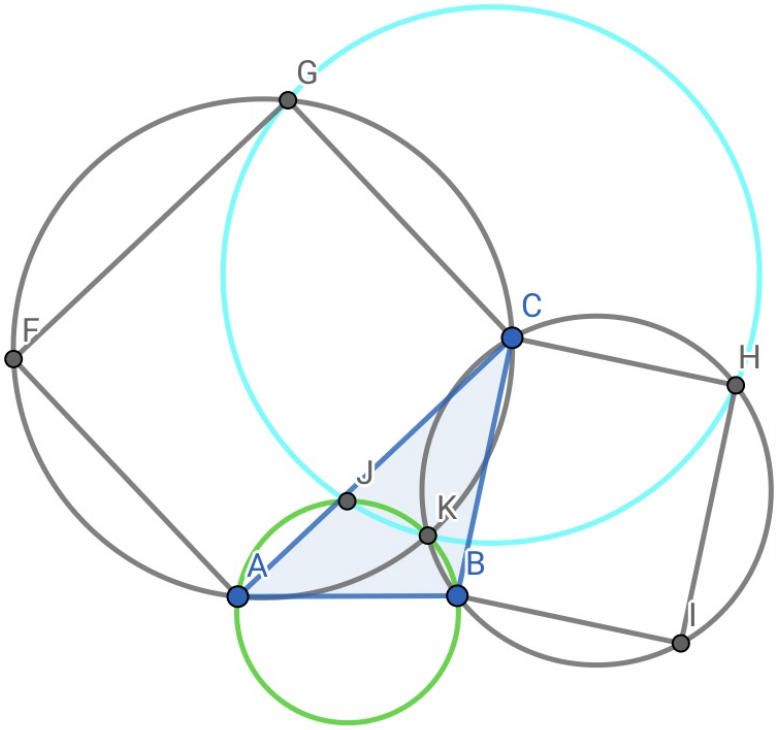

Figure 1. Illustration.