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An Investigation into Fuzzy Wasan Geometry

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Abstract. Wasan geometry is a traditional Japanese branch of mathematics known for its elegant synthetic solutions to problems involving circles, polygons, and other geometric figures. Fuzzy set theory, in contrast, provides a principled way to model uncertainty and partial truth by assigning to each element a membership degree in the unit interval. Fuzzy geometry transfers this idea to classical geometry by introducing graded membership functions for points, lines, and geometric relations. At present, however, the fusion of Wasan geometry with fuzzy geometry has received little systematic attention. To help close this gap, we extend Wasan geometry within the framework of fuzzy sets and study the resulting structure, which we call *Fuzzy Wasan Geometry*.

Keywords. Fuzzy Geometry, Geometry, Fuzzy Wasan Geometry, Wasan Geometry

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1 Preliminaries

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. Throughout this work, we consider only finite structures.

1.1 Fuzzy Geometry

Fuzzy set theory assigns membership values between zero and one to elements, thereby modeling gradual belonging, uncertainty, and partial truth in mathematical contexts [1–3]. Classical geometry studies shapes, sizes, and spatial relations through axioms and constructions, traditionally grounded in Euclidean principles and classical logic [4]. Fuzzy geometry incorporates uncertainty into geometric modeling by assigning graded membership values to points, lines, and spatial relations (cf. [5, 6]). Due to its conceptual importance, fuzzy geometry has continued to attract considerable research attention in recent years [7–9]. Related notions, such as *neutrosophic geometry* [10, 11] and *fuzzy hypergeometry* [12] have also been investigated, and a variety of extensions and applications have been proposed. We now provide a formal definition of fuzzy geometry.

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Definition 1.1 (Fuzzy Set)**.** [1, 13] Let X be a nonempty set. A *fuzzy set* A in X is a function

$$A : X \longrightarrow [0, 1],$$

where $A(x)$ is the *grade of membership* of x in A. Its *support*, *a–level* and *a–cut* are the classical sets

$$\mathrm{Supp}(A) = \{x \in X \mid A(x) > 0\}, \quad A^a = \{x \in X \mid A(x) = a\}, \quad A_a = \{x \in X \mid A(x) \geq a\},$$

respectively.

Example 1.2 (Fuzzy Set)**.** Let $X = \{1, 2, 3, 4, 5\}$. Define a fuzzy set A on X by

$$A = \{(1, 0.1),\ (2, 0.5),\ (3, 1.0),\ (4, 0.7),\ (5, 0.2)\},$$

meaning $A(1) = 0.1$, $A(2) = 0.5$, $A(3) = 1.0$, $A(4) = 0.7$, $A(5) = 0.2$. Thus element 3 belongs fully, element 2 belongs partially, and element 1 belongs only weakly.

Definition 1.3 (Fuzzy Geometry)**.** *Fuzzy Geometry* is a geometric framework in which classical objects such as points, lines, and polygons are replaced by fuzzy sets. Formally, each geometric entity is represented by a membership function

$$\mu : \mathbb{R}^n \longrightarrow [0, 1],$$

where $\mu(x)$ indicates the degree to which a spatial element x belongs to the object. Relations such as incidence, distance, or tangency are also extended to fuzzy forms, allowing the study of geometry under uncertainty.

Example 1.4 (Fuzzy Geometry)**.** Consider the fuzzy disk in $\mathbb{R}^2$ with center $(0, 0)$ and radius $r = 1$. Its membership function is

$$\mu(x, y) = \max\left\{0,\ 1 - \sqrt{x^2 + y^2}\right\}, \qquad (x, y) \in \mathbb{R}^2.$$

Then $\mu(0, 0) = 1$ (full membership at the center), $\mu(1, 0) = 0$ (on the boundary), and $\mu(0.5, 0.5) = 1 - \sqrt{0.5^2 + 0.5^2} \approx 0.293$. This represents a fuzzy circle whose boundary is gradually defined rather than sharp.

1.2 Wasan Geometry

Wasan Geometry studies traditional Japanese geometric problems, often involving circles and polygons, emphasizing synthetic methods and elegant problem-solving approaches (cf. [14–16]).

Definition 1.5 (Wasan Geometry)**.** *Wasan Geometry* refers to the body of geometric knowledge and methods developed in Japan during the Edo period (1603–1868). It is characterized by the use of traditional Japanese mathematics (*Wasan*), which emphasized problems involving circles, ellipses, polygons, and intricate configurations often inscribed on wooden tablets (*sangaku* [17–19]) dedicated at Shinto shrines and Buddhist temples.

Formally, a *Wasan Geometric Framework* can be regarded as a pair

$$\mathcal{W} = (P, \mathcal{R}),$$

where

- P is a finite or infinite set of geometric objects (e.g. points, lines, circles, polygons),
- $\mathcal{R}$ is a family of relations and constructions derived from classical Wasan problem techniques, including circle tangency, area ratios, and nested polygonal or circular figures.

Typical results of Wasan Geometry involve explicit algebraic formulas for lengths, areas, or radii in highly symmetric configurations, usually solved without analytic geometry or modern calculus but through ingenious synthetic and arithmetic arguments.

As a reference for the ideas of Wasan Geometry, we present several concrete examples. Various problems were formulated and studied in Japan during the Edo period.

Example 1.6 (Incircle radius in a right triangle)**.** Let $\triangle ABC$ be right-angled at C, with legs $a = |BC|$, $b = |CA|$, and hypotenuse $c = |AB| = \sqrt{a^2 + b^2}$. Its incircle has radius r. Using the area identity $[ABC] = rs$ with semiperimeter $s = \frac{a+b+c}{2}$ and the right-triangle area $[ABC] = \frac{ab}{2}$,

$$r = \frac{[ABC]}{s} = \frac{\frac{ab}{2}}{\frac{a+b+c}{2}} = \frac{ab}{a+b+c}.$$

Equivalently, in a right triangle one also has

$$r = \frac{a+b-c}{2}.$$

Indeed,

$$\frac{ab}{a+b+c} = \frac{a+b-c}{2} \iff (a+b-c)(a+b+c) = 2ab \iff (a+b)^2 - c^2 = 2ab,$$

which holds since $c^2 = a^2 + b^2$. Thus $r = \dfrac{ab}{a+b+c} = \dfrac{a+b-c}{2}$.

Example 1.7 (Two circles tangent to two perpendicular lines (sangaku-style))**.** Let the two coordinate axes be the perpendicular lines. Consider two circles tangent to both axes with radii r_0 and r ($r \le r_0$). Their centers are (r_0, r_0) and (r, r), and external tangency gives

$$\sqrt{(r_0 - r)^2 + (r_0 - r)^2} = r_0 + r \quad \iff \quad \sqrt{2}\,|r_0 - r| = r_0 + r.$$

If $r \le r_0$, then $\sqrt{2}(r_0 - r) = r_0 + r$, hence

$$(\sqrt{2}+1)r = (\sqrt{2}-1)r_0 \quad \iff \quad r = \frac{\sqrt{2}-1}{\sqrt{2}+1}\, r_0 = \frac{(\sqrt{2}-1)^2}{2-1}\, r_0 = (3 - 2\sqrt{2})\, r_0.$$

So the smaller circle has radius $r = (3-2\sqrt{2})\, r_0$ (and the larger solution is $(3+2\sqrt{2})\, r_0$).

Example 1.8 (Square inscribed in a semicircle)**.** Let a semicircle of radius R lie above the x-axis with center at the origin. Place a square of side s with its base on the diameter and centered at the origin; its top vertices $(\pm s/2, s)$ lie on the semicircle $x^2 + y^2 = R^2$:

$$\left(\frac{s}{2}\right)^2 + s^2 = R^2 \iff \frac{5}{4}s^2 = R^2 \iff s = \frac{2}{\sqrt{5}}\, R.$$

Thus the side length of the inscribed square is $s = 2R/\sqrt{5}$.

Example 1.9 (Circle tangent to two equal circles and a line)**.** Two equal circles of radius R rest on the line $y = 0$ with centers at $(-R, R)$ and (R, R). Find the radius r of the circle tangent to both and to $y = 0$ above them. By symmetry, its center is $(0, r)$ and tangency to $y = 0$ forces center height r. Tangency to the left circle yields

$$\begin{aligned}\sqrt{(0+R)^2 + (r-R)^2} = R + r &\iff R^2 + (r-R)^2 = (R+r)^2 \\ &\iff r^2 - 2Rr + 2R^2 = r^2 + 2Rr + R^2 \\ &\iff -4Rr = -R^2\end{aligned}$$

$$\iff \quad r = \frac{R}{4}.$$

Hence the desired radius is $r = R/4$.

2 Result: Fuzzy Wasan Geometry

Fuzzy Wasan Geometry extends Wasan Geometry by introducing fuzzy sets, modeling uncertain lengths, radii, or incidences with graded membership values.

Definition 2.1 (Fuzzy Wasan Geometry)**.** A *Fuzzy Wasan Geometry* is an extension of classical Wasan Geometry in which geometric objects and their relations are endowed with degrees of membership, allowing the treatment of uncertainty and vagueness in traditional Wasan problems.

Formally, let P denote the set of geometric primitives considered in Wasan Geometry (e.g. points, lines, circles, polygons). Define

$$\mathcal{F}(P) = \{\, \mu : P \to [0,1] \,\}$$

as the collection of *fuzzy geometric objects*, where $\mu(x)$ expresses the degree to which $x \in P$ belongs to a fuzzy geometric configuration.

A *Fuzzy Wasan Geometry* is the pair

$$\mathcal{FW} = (\mathcal{F}(P),\, \mathcal{R}_f),$$

where

- $\mathcal{F}(P)$ is the set of fuzzy geometric entities (fuzzy points, fuzzy lines, fuzzy circles, etc.),
- $\mathcal{R}_f$ is a family of fuzzy relations and constructions adapted from Wasan Geometry (e.g. fuzzy tangency between circles, fuzzy concurrency of lines, fuzzy area ratios of polygons).

Thus, a Fuzzy Wasan Geometry generalizes the classical Wasan framework by incorporating fuzzy set theory, enabling the study of Wasan-style geometric problems under uncertainty or partial membership.

Example 2.2 (Fuzzy incircle in a right triangle)**.** For a triangular fuzzy number $\tilde{x} = (x_L, x_M, x_U)$ with $0 \le x_L \le x_M \le x_U$, the α–cut is

$$[\tilde{x}]_\alpha = \big[x_L + \alpha(x_M - x_L),\ x_U - \alpha(x_U - x_M)\big] \qquad (0 \le \alpha \le 1).$$

Let the legs be fuzzy lengths $\tilde{a} = (6.8, 7.0, 7.2)$ and $\tilde{b} = (9.8, 10.0, 10.2)$ (units). For each α,

$$[a]_\alpha = [a_L(\alpha), a_U(\alpha)], \quad a_L(\alpha) = 6.8 + 0.2\alpha,\ a_U(\alpha) = 7.2 - 0.2\alpha,$$
$$[b]_\alpha = [b_L(\alpha), b_U(\alpha)], \quad b_L(\alpha) = 9.8 + 0.2\alpha,\ b_U(\alpha) = 10.2 - 0.2\alpha.$$

With $c(a,b) = \sqrt{a^2 + b^2}$ increasing in $a, b > 0$,

$$[c]_\alpha = \big[\sqrt{a_L(\alpha)^2 + b_L(\alpha)^2},\ \sqrt{a_U(\alpha)^2 + b_U(\alpha)^2}\big].$$

The inradius in a right triangle is

$$r(a,b) = \frac{ab}{a + b + \sqrt{a^2 + b^2}},$$

which is increasing in $a, b > 0$. Hence

$$[\tilde{r}]_\alpha = \big[r\big(a_L(\alpha), b_L(\alpha)\big),\ r\big(a_U(\alpha), b_U(\alpha)\big)\big].$$

Numerically,

$$[\tilde{r}]_0 \approx [2.3359409795,\ 2.4574043860], \qquad [\tilde{r}]_1 \approx [2.3967221921,\ 2.3967221921].$$

Example 2.3 (Fuzzy circle tangent to three mutually tangent circles (Descartes))**.** Let three mutually tangent circles have fuzzy radii

$$\tilde{r}_1 = (2.9, 3.0, 3.1), \quad \tilde{r}_2 = (4.9, 5.0, 5.1), \quad \tilde{r}_3 = (6.9, 7.0, 7.1).$$

Write curvatures $k_i = 1/r_i$. For $[r_i]_\alpha = [r_{i,L}, r_{i,U}]$ (with $r_{i,L} \le r_{i,U}$),

$$[k_i]_\alpha = \Big[\frac{1}{r_{i,U}},\ \frac{1}{r_{i,L}}\Big],$$

since $r \mapsto 1/r$ is decreasing on $(0, \infty)$. The inner Apollonian curvature is

$$k_4^+ = k_1 + k_2 + k_3 + 2\sqrt{k_1k_2 + k_2k_3 + k_3k_1},$$

which is increasing in each $k_i > 0$. Therefore

$$[k_4^+]_\alpha = \big[k_4^+(k_{1,L}, k_{2,L}, k_{3,L}),\ k_4^+(k_{1,U}, k_{2,U}, k_{3,U})\big], \qquad [r_4]_\alpha = \Big[\frac{1}{k_{4,U}^+},\ \frac{1}{k_{4,L}^+}\Big].$$

For the above data,

$$[r_4]_0 \approx [0.6810876312,\ 0.7153662761], \qquad [r_4]_1 \approx [0.6982657902,\ 0.6982657902].$$

Example 2.4 (Fuzzy area ratio of nested squares)**.** Let an outer square have fuzzy side $\tilde{s}_P = (9.8, 10.0, 10.3)$ and an inner, concentric square have fuzzy side $\tilde{s}_Q = (4.8, 5.0, 5.2)$. For each α,

$$[s_P]_\alpha = [s_{P,L}, s_{P,U}], \qquad [s_Q]_\alpha = [s_{Q,L}, s_{Q,U}].$$

Areas are $A_P = s_P^2$, $A_Q = s_Q^2$, so (monotone on $[0, \infty)$)

$$[A_P]_\alpha = [s_{P,L}^2,\ s_{P,U}^2], \qquad [A_Q]_\alpha = [s_{Q,L}^2,\ s_{Q,U}^2].$$

The ratio $\rho = A_P/A_Q$ is increasing in A_P and decreasing in A_Q, hence

$$[\tilde{\rho}]_\alpha = \Big[\frac{s_{P,L}^2}{s_{Q,U}^2},\ \frac{s_{P,U}^2}{s_{Q,L}^2}\Big].$$

Numerically,

$$[\tilde{\rho}]_0 \approx [3.5517751479,\ 4.6046006944], \qquad [\tilde{\rho}]_1 \approx [4.0000000000,\ 4.0000000000].$$

Example 2.5 (Fuzzy concurrency of cevians via fuzzy lines)**.** Model three cevians as fuzzy lines in $\mathbb{R}^2$ with Gaussian memberships

$$\mu_i(p) = \exp\Big(-\frac{\operatorname{dist}(p, \ell_i)^2}{\sigma_i^2}\Big), \qquad i = 1, 2, 3.$$

Take

$$\ell_1 :\ x = 0, \qquad \ell_2 :\ y = 0, \qquad \ell_3 :\ x + y = 1 + \varepsilon,$$

with $\sigma_1 = \sigma_2 = \sigma_3 = \sigma > 0$ and $\varepsilon \ge 0$ (perfect concurrency at $\varepsilon = 0$). Define the fuzzy concurrency degree

$$\gamma = \sup_{p \in \mathbb{R}^2} \min\big\{\mu_1(p), \mu_2(p), \mu_3(p)\big\}.$$

By symmetry, the maximizer lies on $p = (t, t)$; minimizing the worst (scaled) distance yields

$$t = \frac{|2t - (1 + \varepsilon)|}{\sqrt{2}} \quad \Longrightarrow \quad t^\star = \frac{1 + \varepsilon}{2 + \sqrt{2}}.$$

At $p^\star = (t^\star, t^\star)$ the three distances are equal, hence

$$\gamma = \exp\Big(-\frac{(t^\star)^2}{\sigma^2}\Big), \qquad t^\star = \frac{1+\varepsilon}{2+\sqrt{2}}.$$

For $\varepsilon = 0.1$ and $\sigma = 1$,

$$t^\star \approx 0.322177, \qquad \gamma \approx \exp(-0.322177^2) \approx 0.90140.$$

When $\varepsilon = 0$ (exact concurrency) and $\sigma \to \infty$, one has $\gamma \to 1$, recovering the crisp case.

Notation 2.6 (Crisp-to-fuzzy lifting of relations)**.** *Let P be the set of Wasan primitives (points, lines, circles, polygons, …). For any k-ary crisp relation $R \subseteq P^k$ (e.g. incidence, tangency, concurrency), define its* fuzzy lift $\widehat{R} : \mathcal{F}(P)^k \to [0,1]$ *by*

$$\widehat{R}(\mu_1, \dots, \mu_k) = \sup_{(x_1,\dots,x_k)\in R} \min\{\mu_1(x_1), \dots, \mu_k(x_k)\},$$

where each $\mu_i : P \to [0,1]$ is a fuzzy object. For a classical object $x \in P$, write $\delta_x : P \to \{0,1\}$ for the Dirac (indicator) membership $\delta_x(y) = 1$ if $y = x$ and 0 otherwise.

Lemma 2.7 (Monotonicity of the lift)**.** *If $\mu_i \le \nu_i$ pointwise on P for all $i = 1, \dots, k$, then*

$$\widehat{R}(\mu_1, \dots, \mu_k) \;\le\; \widehat{R}(\nu_1, \dots, \nu_k).$$

Proof. Fix $(x_1, \dots, x_k) \in R$. Since $\mu_i(x_i) \le \nu_i(x_i)$ for each i, we have

$$\min_{1\le i\le k} \mu_i(x_i) \;\le\; \min_{1\le i\le k} \nu_i(x_i).$$

Taking sup over $(x_1, \dots, x_k) \in R$ preserves the inequality, giving the claim. □

Theorem 2.8 (Fuzzy Wasan Geometry is a Fuzzy Geometry)**.** *Let $\mathcal{FW} = (\mathcal{F}(P), \mathcal{R}_f)$ where $\mathcal{R}_f$ consists of the fuzzy lifts $\widehat{R}$ (Notation 2.6) of the classical Wasan relations $R \in \mathcal{R}$. Then $\mathcal{FW}$ is a* Fuzzy Geometry *in the sense that every geometric entity is a membership function on an underlying space and every classical relation is extended to a fuzzy relation with values in $[0,1]$, monotone in each argument.*

Proof. Take the underlying space $X := P$ (the universe of Wasan primitives). Every fuzzy entity is a function $\mu : X \to [0,1]$, i.e. an element of $\mathcal{F}(P)$. For each classical k-ary relation $R \subseteq P^k$, the lift $\widehat{R}$ maps k fuzzy entities to $[0,1]$ by a sup∘min aggregator, hence is a well-defined fuzzy relation. By Lemma 2.7 these fuzzy relations are monotone in each coordinate. Thus $\big(\mathcal{F}(P), \{\widehat{R}\}_{R\in\mathcal{R}}\big)$ satisfies the standard axioms of a fuzzy geometric framework: objects are memberships on X and relations evaluate to grades in $[0,1]$ in a monotone way. If one prefers the ambient space $\mathbb{R}^n$, choose an injective encoding $e : P \hookrightarrow \mathbb{R}^n$ and push each μ forward by $\mu'(y) := \mu(e^{-1}(y))$ for $y \in e(P)$ and 0 otherwise; the construction is equivalent. □

Theorem 2.9 (Fuzzy generalizes Wasan via a crisp embedding)**.** *Define $\iota : P \to \mathcal{F}(P)$ by $\iota(x) := \delta_x$. Then for any k-ary classical relation $R \subseteq P^k$ and any $x_1, \dots, x_k \in P$,*

$$\widehat{R}\big(\delta_{x_1}, \dots, \delta_{x_k}\big) = \begin{cases} 1, & \text{if } (x_1, \dots, x_k) \in R, \\ 0, & \text{otherwise.} \end{cases}$$

Consequently, $(P, \mathcal{R})$ embeds as the $\{0,1\}$-valued substructure of $\mathcal{FW}$; in particular, Fuzzy Wasan Geometry strictly generalizes Wasan Geometry.

Proof. Suppose $(x_1, \ldots, x_k) \in R$. Then in the supremum defining $\widehat{R}$ the tuple $(x_1, \ldots, x_k)$ contributes the term

$$\min\{\delta_{x_1}(x_1), \ldots, \delta_{x_k}(x_k)\} = \min\{1, \ldots, 1\} = 1,$$

so $\widehat{R}(\delta_{x_1}, \ldots, \delta_{x_k}) \geq 1$. Since values lie in $[0, 1]$, equality $= 1$ follows.

Conversely, if $(x_1, \ldots, x_k) \notin R$, take any $(y_1, \ldots, y_k) \in R$. At least one index i satisfies $y_i \neq x_i$, hence $\delta_{x_i}(y_i) = 0$, so the corresponding $\min\{\delta_{x_1}(y_1), \ldots, \delta_{x_k}(y_k)\} = 0$. Therefore every term in the supremum is 0, giving $\widehat{R}(\delta_{x_1}, \ldots, \delta_{x_k}) = 0$. Injectivity of ι is obvious, and the relation-preservation calculation above shows that ι is a strong homomorphism into the $\{0, 1\}$-valued part of $\mathcal{FW}$. Hence Wasan Geometry is isomorphic to the crisp substructure of Fuzzy Wasan Geometry, proving generalization. □

3 Conclusion

In this paper, we extended Wasan geometry within the framework of fuzzy sets and investigated the resulting structure, which we call *Fuzzy Wasan Geometry*. As future work, we hope to see (i) the formulation and solution of concrete Wasan-geometry problems in this setting, and (ii) the development of richer extensions based on more advanced uncertainty frameworks, including Intuitionistic Fuzzy Sets [20, 21], Spherical Fuzzy Sets [22], HyperFuzzy Sets [23, 24], Neutrosophic Sets [25–27], and Plithogenic Sets [28, 29]. More broadly, we expect that research on a wide range of Wasan-geometry problems will continue to advance by incorporating such extensions—including fuzzy and neutrosophic approaches—to capture uncertainty and multi-valued reasoning in a systematic way.

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Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate

referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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