International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 10, 2025, pp. 302-305.
Received June 15, 2025. Published on-line July 16, 2025
web: http://www.journal-1.eu/
©The Author(s) This article is published with open access[1].

A Point on the Circumcircle Related to the Cevian Triangle

Benjamin Lee Warren[a] [2]
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

Mathematics Subject Classification (2010). 51N20.

1. Introduction

We justify the claim made in Euclid #8563.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let P be a point in the plane.
Let $A'B'C'$ be the cevian triangle of P.
Let the line passing through A' which is orthogonal to line BC intersect lines AC and AB at points A_b and A_c respectively.
Define B_a, B_c, C_a, and C_b cyclically.
Then circles ABC, AB_aC_a, BA_bC_b, and CA_cB_c are concurrent.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
P has coordinates (x_P, y_P).
A' has coordinates
$((x_P * y_C)/(-x_C * y_P + y_P + x_P * y_C), (y_C * y_P)/(-x_C * y_P + y_P + x_P * y_C))$.
B' has coordinates
$((x_C * y_P)/(x_C * y_P - x_P * y_C + y_C), (y_C * y_P)/(x_C * y_P - x_P * y_C + y_C))$.
C' has coordinates $((x_C * y_P - x_P * y_C)/(y_P - y_C), 0)$.
A_b has coordinates

[1]This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.
[2]Corresponding author

$(-(x_C*y_C*(y_C*y_P+(x_C-1)*x_P))/((y_C^2+(x_C-1)*x_C)*((x_C-1)*y_P-x_P*y_C)), -(y_C^2*(y_C*y_P+(x_C-1)*x_P))/((y_C^2+(x_C-1)*x_C)*((x_C-1)*y_P-x_P*y_C))).$
A_c has coordinates
$((y_C*(y_C*y_P+x_C*x_P-x_P))/((x_C-1)*(-x_C*y_P+y_P+x_P*y_C)), 0).$
B_a has coordinates
$((((2*x_C-1)*y_C^2+(x_C-1)*x_C^2)*y_P-(x_P-1)*y_C^3)/((y_C^2+(x_C-1)*x_C)*(x_C*y_P-(x_P-1)*y_C)), (y_C^2*(y_C*y_P+x_C*(x_P-1)))/((y_C^2+(x_C-1)*x_C)*(x_C*y_P-(x_P-1)*y_C))).$
B_c has coordinates
$((y_P*(x_C^2+y_C^2))/(x_C*(y_C+x_C*y_P-x_P*y_C)), 0).$
C_a has coordinates
$((x_C*y_P-x_P*y_C)/(y_P-y_C), ((x_C*y_P-x_P*y_C)/(y_P-y_C)-1)*y_C/(x_C-1)).$
C_b has coordinates
$((x_C*y_P-x_P*y_C)/(y_P-y_C), ((x_C*y_P-x_P*y_C)/(y_P-y_C))*y_C/x_C).$
Circle ABC is given by the equation
$(x-1/2)^2+(y-((y_C^2+(x_C-1)*x_C)/(2*y_C)))^2-1/4-((y_C^2+(x_C-1)*x_C)/(2*y_C))^2=0.$
Circle AB_aC_a is given by the equation
$(x-(-((x_C-1)*(y_C^2*(y_C^2+x_C^2-3*x_C+1)-(x_C-1)*x_C^2)*y_P^2-y_C*(y_C^2*((x_P-1)*y_C^2-2*(x_C-1)*x_P-(x_C-2)*(x_C-1))-(x_C-1)^2*x_C^2*x_P)*y_P-(x_P-1)*(x_C*x_P-1)*y_C^4-(x_C-1)^2*x_C*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2+(y-(((x_C-1)*x_C*(y_C^2*(2*y_C^2+(x_C-1)*(3*x_C-1))+(x_C-1)^2*x_C^2)*y_P^2-y_C*(3*x_C*x_P*y_C^4-2*x_P*y_C^4-2*x_C*y_C^4+y_C^4+4*x_C^3*x_P*y_C^2-9*x_C^2*x_P*y_C^2+6*x_C*x_P*y_C^2-x_P*y_C^2-2*x_C^3*y_C^2+4*x_C^2*y_C^2-2*x_C*y_C^2+x_C^5*x_P-3*x_C^4*x_P+3*x_C^3*x_P-x_C^2*x_P)*y_P+(x_P-1)^2*y_C^6+(x_C-1)*(x_P-1)*((x_C-1)*x_P-x_C)*y_C^4)/(2*(x_C-1)*y_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-((-((x_C-1)*(y_C^2*(y_C^2+x_C^2-3*x_C+1)-(x_C-1)*x_C^2)*y_P^2-y_C*(y_C^2*((x_P-1)*y_C^2-2*(x_C-1)*x_P-(x_C-2)*(x_C-1))-(x_C-1)^2*x_C^2*x_P)*y_P-(x_P-1)*(x_C*x_P-1)*y_C^4-(x_C-1)^2*x_C*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-((((x_C-1)*x_C*(y_C^2*(2*y_C^2+(x_C-1)*(3*x_C-1))+(x_C-1)^2*x_C^2)*y_P^2-y_C*(3*x_C*x_P*y_C^4-2*x_P*y_C^4-2*x_C*y_C^4+y_C^4+4*x_C^3*x_P*y_C^2-9*x_C^2*x_P*y_C^2+6*x_C*x_P*y_C^2-x_P*y_C^2-2*x_C^3*y_C^2+4*x_C^2*y_C^2-2*x_C*y_C^2+x_C^5*x_P-3*x_C^4*x_P+3*x_C^3*x_P-x_C^2*x_P)*y_P+(x_P-1)^2*y_C^6+(x_C-1)*(x_P-1)*((x_C-1)*x_P-x_C)*y_C^4)/(2*(x_C-1)*y_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2=0.$
Circle BA_bC_b is given by the equation
$(x-(-(x_C*(y_C^2*(y_C^2+x_C^2-x_C+1)-(x_C-1)^2*x_C)*y_P^2-y_C*(x_P*y_C^4-x_C^2*y_C^2+x_C*y_C^2-x_C^4*x_P+x_C^2*x_P-x_C^4+2*x_C^3-x_C^2)*y_P-x_P*((x_C-1)*x_P+x_C)*y_C^4-(x_C-1)*x_C^2*x_P*(x_P+1)*y_C^2)/(2*x_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2+(y-((((2*x_C^2-2*x_C)*y_C^4+(3*x_C^4-5*x_C^3+2*x_C^2)*y_C^2+x_C^6-3*x_C^5+3*x_C^4-x_C^3)*y_P^2+(((1-3*x_C)*x_P+x_C)*y_C^5+((3*x_C^2-4*x_C^3)*x_P+2*x_C^3-x_C^2)*y_C^3+((-x_C^5+2*x_C^4-x_C^3)*x_P+x_C^5-2*x_C^4+x_C^3)*y_C)*y_P+x_P^2*y_C^6+(x_C^2*x_P^2-x_C*x_P)*y_C^4)/(2*x_C*y_C*(y_C^2+x_C^2-x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2-(1-(-(x_C*(y_C^2*(y_C^2+x_C^2-x_C+1)-(x_C-1)^2*x_C)*y_P^2-y_C*(x_P*y_C^4-x_C^2*y_C^2+x_C*y_C^2-x_C^4*x_P+x_C^2*x_P-x_C^4+2*x_C^3-x_C^2)*y_P-x_P*((x_C-1)*x_P+x_C)*y_C^4-(x_C-1)*x_C^2*x_P*(x_P+1)*y_C^2)/(2*x_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2-((((2*x_C^2-2*x_C)*y_C^4+(3*x_C^4-5*x_C^3+2*x_C^2)*y_C^2+x_C^6-3*x_C^5+3*x_C^4-x_C^3)*y_P^2+(((1-3*x_C)*x_P+x_C)*y_C^5+((3*x_C^2-4*x_C^3)*x_P+2*x_C^3-x_C^2)*y_C^3+((-x_C^5+2*x_C^4-x_C^3)*x_P+x_C^5-2*x_C^4+x_C^3)*y_C)*y_P+x_P^2*y_C^6+(x_C^2*x_P^2-x_C*x_P)*y_C^4)/(2*x_C*y_C*(y_C^2+x_C^2-x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2=0.$

Circle CA_cB_c is given by the equation
$(x-(-(((2*x_C-1)*y_C^2-x_C^4+2*x_C^3-x_C^2)*y_P^2+((x_C-x_P)*y_C^3+(2*x_C^3-2*x_C^2)*x_P*y_C)*y_P+((x_C-x_C^2)*x_P^2+(x_C^2-x_C)*x_P)*y_C^2)/(2*(x_C-1)*x_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P+(1-x_P)*y_C))))^2+(y-(-((y_C^4-(x_C-1)*x_C*(x_C^2-x_C+1)*y_C^2-x_C^6+3*x_C^5-3*x_C^4+x_C^3)*y_P^2+((2*x_C-1)*x_P-x_C)*y_C*((x_C^2-x_C+1)*y_C^2+(x_C-1)^2*x_C^2)*y_P-(x_C-1)*x_C*(x_P-1)*x_P*y_C^4-(x_C-1)^2*x_C^2*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*x_C*y_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-(x_C-(-(((2*x_C-1)*y_C^2-x_C^4+2*x_C^3-x_C^2)*y_P^2+((x_C-x_P)*y_C^3+(2*x_C^3-2*x_C^2)*x_P*y_C)*y_P+((x_C-x_C^2)*x_P^2+(x_C^2-x_C)*x_P)*y_C^2)/(2*(x_C-1)*x_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P+(1-x_P)*y_C))))^2-(y_C-(-((y_C^4-(x_C-1)*x_C*(x_C^2-x_C+1)*y_C^2-x_C^6+3*x_C^5-3*x_C^4+x_C^3)*y_P^2+((2*x_C-1)*x_P-x_C)*y_C*((x_C^2-x_C+1)*y_C^2+(x_C-1)^2*x_C^2)*y_P-(x_C-1)*x_C*(x_P-1)*x_P*y_C^4-(x_C-1)^2*x_C^2*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*x_C*y_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P-(x_P-1)*y_C))))^2=0.$
Consider the point whose coordinates are given by
$(((y_C*y_P+(x_C-1)*x_P)*(x_C*y_C*y_P-(x_P-x_C)*y_C^2+x_C^3-x_C^2))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2), (y_C*(y_C*y_P+x_C*(x_P-1))*(y_C*y_P+(x_C-1)*x_P))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2))$.
We verify that this point lies on all four circles since we calculate
$(((y_C*y_P+(x_C-1)*x_P)*(x_C*y_C*y_P-(x_P-x_C)*y_C^2+x_C^3-x_C^2))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-1/2)^2+((y_C*(y_C*y_P+x_C*(x_P-1))*(y_C*y_P+(x_C-1)*x_P))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-((y_C^2+(x_C-1)*x_C)/(2*y_C)))^2-1/4-((y_C^2+(x_C-1)*x_C)/(2*y_C))^2=0,$
$(((y_C*y_P+(x_C-1)*x_P)*(x_C*y_C*y_P-(x_P-x_C)*y_C^2+x_C^3-x_C^2))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-(-((x_C-1)*(y_C^2*(y_C^2+x_C^2-3*x_C+1)-(x_C-1)*x_C^2)*y_P^2-y_C*(y_C^2*((x_P-1)*y_C^2-2*(x_C-1)*x_P-(x_C-2)*(x_C-1))-(x_C-1)^2*x_C^2*x_P)*y_P-(x_P-1)*(x_C*x_P-1)*y_C^4-(x_C-1)^2*x_C*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2+((y_C*(y_C*y_P+x_C*(x_P-1))*(y_C*y_P+(x_C-1)*x_P))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-(((x_C-1)*x_C*(y_C^2*(2*y_C^2+(x_C-1)*(3*x_C-1))+(x_C-1)^2*x_C^2)*y_P^2-y_C*(3*x_C*x_P*y_C^4-2*x_P*y_C^4-2*x_C*y_C^4+y_C^4+4*x_C^3*x_P*y_C^2-9*x_C^2*x_P*y_C^2+6*x_C*x_P*y_C^2-x_P*y_C^2-2*x_C^3*y_C^2+4*x_C^2*y_C^2-2*x_C*y_C^2+x_C^5*x_P-3*x_C^4*x_P+3*x_C^3*x_P-x_C^2*x_P)*y_P+(x_P-1)^2*y_C^6+(x_C-1)*(x_P-1)*((x_C-1)*x_P-x_C)*y_C^4)/(2*(x_C-1)*y_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-((-((x_C-1)*(y_C^2*(y_C^2+x_C^2-3*x_C+1)-(x_C-1)*x_C^2)*y_P^2-y_C*(y_C^2*((x_P-1)*y_C^2-2*(x_C-1)*x_P-(x_C-2)*(x_C-1))-(x_C-1)^2*x_C^2*x_P)*y_P-(x_P-1)*(x_C*x_P-1)*y_C^4-(x_C-1)^2*x_C*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-((((x_C-1)*x_C*(y_C^2*(2*y_C^2+(x_C-1)*(3*x_C-1))+(x_C-1)^2*x_C^2)*y_P^2-y_C*(3*x_C*x_P*y_C^4-2*x_P*y_C^4-2*x_C*y_C^4+y_C^4+4*x_C^3*x_P*y_C^2-9*x_C^2*x_P*y_C^2+6*x_C*x_P*y_C^2-x_P*y_C^2-2*x_C^3*y_C^2+4*x_C^2*y_C^2-2*x_C*y_C^2+x_C^5*x_P-3*x_C^4*x_P+3*x_C^3*x_P-x_C^2*x_P)*y_P+(x_P-1)^2*y_C^6+(x_C-1)*(x_P-1)*((x_C-1)*x_P-x_C)*y_C^4)/(2*(x_C-1)*y_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*(x_C*y_P-(x_P-1)*y_C))))^2=0,$
$(((y_C*y_P+(x_C-1)*x_P)*(x_C*y_C*y_P-(x_P-x_C)*y_C^2+x_C^3-x_C^2))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-(-(x_C*(y_C^2*(y_C^2+x_C^2-x_C+1)-(x_C-1)^2*x_C)*y_P^2-y_C*(x_P*y_C^4-x_C^2*y_C^2+x_C*y_C^2-x_C^4*x_P+x_C^2*x_P-x_C^4+2*x_C^3-x_C^2)*y_P-x_P*((x_C-1)*x_P+x_C)*y_C^4-(x_C-1)*x_C^2*x_P*(x_P+1)*y_C^2)/(2*x_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2+((y_C*(y_C*y_P+x_C*(x_P-1))*(y_C*y_P+(x_C-1)*x_P))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-((((2*x_C^2-2*x_C)*y_C^4+(3*x_C^4-5*x_C^3+2*x_C^2)*y_C^2+x_C^6-3*x_C^5+3*x_C^4-x_C^3)*y_P^2+((1-$

$3*x_C)*x_P+x_C)*y_C^5+((3*x_C^2-4*x_C^3)*x_P+2*x_C^3-x_C^2)*y_C^3+((-x_C^5+2*x_C^4-x_C^3)*x_P+x_C^5-2*x_C^4+x_C^3)*y_C)*y_P+x_P^2*y_C^6+(x_C^2*x_P^2-x_C*x_P)*y_C^4)/(2*x_C*y_C*(y_C^2+x_C^2-x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2-(1-(-(x_C*(y_C^2*(y_C^2+x_C^2-x_C+1)-(x_C-1)^2*x_C)*y_P^2-y_C*(x_P*y_C^4-x_C^2*y_C^2+x_C*y_C^2-x_C^4*x_P+x_C^2*x_P-x_C^4+2*x_C^3-x_C^2)*y_P-x_P*((x_C-1)*x_P+x_C)*y_C^4-(x_C-1)*x_C^2*x_P*(x_P+1)*y_C^2)/(2*x_C*(y_C^2+(x_C-1)*x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2-(((((2*x_C^2-2*x_C)*y_C^4+(3*x_C^4-5*x_C^3+2*x_C^2)*y_C^2+x_C^6-3*x_C^5+3*x_C^4-x_C^3)*y_P^2+(((1-3*x_C)*x_P+x_C)*y_C^5+((3*x_C^2-4*x_C^3)*x_P+2*x_C^3-x_C^2)*y_C^3+((-x_C^5+2*x_C^4-x_C^3)*x_P+x_C^5-2*x_C^4+x_C^3)*y_C)*y_P+x_P^2*y_C^6+(x_C^2*x_P^2-x_C*x_P)*y_C^4)/(2*x_C*y_C*(y_C^2+x_C^2-x_C)*(y_P-y_C)*((x_C-1)*y_P-x_P*y_C))))^2 = 0,$
$(((y_C*y_P+(x_C-1)*x_P)*(x_C*y_C*y_P-(x_P-x_C)*y_C^2+x_C^3-x_C^2))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-(-(((2*x_C-1)*y_C^2-x_C^4+2*x_C^3-x_C^2)*y_P^2+((x_C-x_P)*y_C^3+(2*x_C^3-2*x_C^2)*x_P*y_C)*y_P+((x_C-x_C^2)*x_P^2+(x_C^2-x_C)*x_P)*y_C^2)/(2*(x_C-1)*x_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P+(1-x_P)*y_C))))^2+((y_C*(y_C*y_P+x_C*(x_P-1))*(y_C*y_P+(x_C-1)*x_P))/(y_C^2*y_P^2+2*(x_C-1)*x_C*y_C*y_P+(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)-(-((y_C^4-(x_C-1)*x_C*(x_C^2-x_C+1)*y_C^2-x_C^6+3*x_C^5-3*x_C^4+x_C^3)*y_P^2+((2*x_C-1)*x_P-x_C)*y_C*((x_C^2-x_C+1)*y_C^2+(x_C-1)^2*x_C^2)*y_P-(x_C-1)*x_C*(x_P-1)*x_P*y_C^4-(x_C-1)^2*x_C^2*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*x_C*y_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P-(x_P-1)*y_C))))^2-(x_C-(-(((2*x_C-1)*y_C^2-x_C^4+2*x_C^3-x_C^2)*y_P^2+((x_C-x_P)*y_C^3+(2*x_C^3-2*x_C^2)*x_P*y_C)*y_P+((x_C-x_C^2)*x_P^2+(x_C^2-x_C)*x_P)*y_C^2)/(2*(x_C-1)*x_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P+(1-x_P)*y_C))))^2-(y_C-(-((y_C^4-(x_C-1)*x_C*(x_C^2-x_C+1)*y_C^2-x_C^6+3*x_C^5-3*x_C^4+x_C^3)*y_P^2+((2*x_C-1)*x_P-x_C)*y_C*((x_C^2-x_C+1)*y_C^2+(x_C-1)^2*x_C^2)*y_P-(x_C-1)*x_C*(x_P-1)*x_P*y_C^4-(x_C-1)^2*x_C^2*(x_P-1)*x_P*y_C^2)/(2*(x_C-1)*x_C*y_C*((x_C-1)*y_P-x_P*y_C)*(x_C*y_P-(x_P-1)*y_C))))^2 = 0$
as desired. □

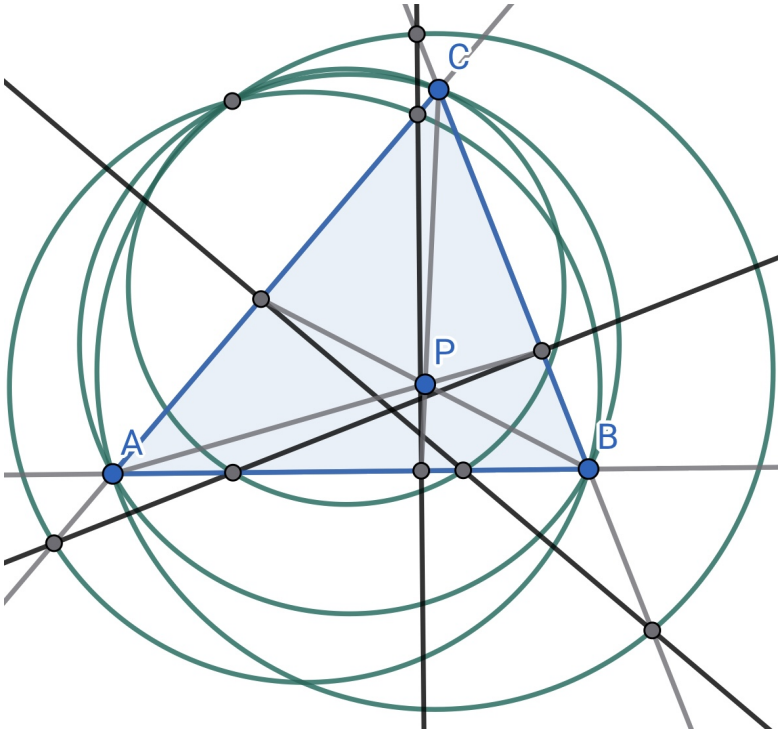

Figure 1. Illustration.