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A Circle of Triangle Centroids

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1. Introduction

We present and prove the existence of a certain circle in the context of the pedal triangle.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be a point in the plane.
Let E, F, and G be the vertices of the pedal triangle of D on lines AB, AC, and BC respectively.
Let H, I, and J be the vertices of a triangle homothetic with EFG about D on lines DE, DF, and DG respectively.
Let α, β, and γ be the circles whose diameters are AJ, BI, and CH respectively.
Let α intersect lines AB and AC at K and L respectively.
Let β intersect lines AB and BC at M and N respectively.
Let γ intersect lines AC and BC at O and P respectively.
Then the centroids of ABC, ANP, BLO, and CKM are concyclic.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates (x_D, y_D).
E has coordinates $(x_D, 0)$.
F has coordinates

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$((x_C * (x_C * x_D + y_C * y_D))/(x_C^2 + y_C^2), (y_C * (x_C * x_D + y_C * y_D))/(x_C^2 + y_C^2))$.
G has coordinates
$((y_C * ((x_C - 1) * y_D + y_C) + (x_C - 1)^2 * x_D)/(y_C^2 + (x_C - 1)^2), (y_C * (y_C * y_D + (x_C - 1) * x_D - x_C + 1))/(y_C^2 + (x_C - 1)^2))$.
H has coordinates (x_D, y_H).
I has coordinates
$(x_D - (((x_C * (y_C * y_D + x_C * x_D))/(y_C^2 + x_C^2) - x_D) * (y_H - y_D))/y_D, (x_C * ((x_C * (y_C * y_D + x_C * x_D))/(y_C^2 + x_C^2) - x_D) * (y_H - y_D))/(y_C * y_D) + y_D)$.
J has coordinates
$(x_D - (((y_C * ((x_C - 1) * y_D + y_C) + (x_C - 1)^2 * x_D)/(y_C^2 + (x_C - 1)^2) - x_D) * (y_H - y_D))/y_D, y_D - ((1 - x_C) * ((y_C * ((x_C - 1) * y_D + y_C) + (x_C - 1)^2 * x_D)/(y_C^2 + (x_C - 1)^2) - x_D) * (y_H - y_D))/(y_C * y_D))$.
α is given by the equation
$(x-(x_D-(((y_C*((x_C-1)*y_D+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)-x_D)*(y_H-y_D))/y_D)/2)^2+(y-(y_D-((1-x_C)*((y_C*((x_C-1)*y_D+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)-x_D)*(y_H-y_D))/(y_C*y_D))/2)^2-((x_D-(((y_C*((x_C-1)*y_D+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)-x_D)*(y_H-y_D))/y_D)/2)^2-((y_D-((1-x_C)*((y_C*((x_C-1)*y_D+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)-x_D)*(y_H-y_D))/(y_C*y_D))/2)^2 = 0$.
β is given by the equation
$(x-((x_D-(((x_C*(y_C*y_D+x_C*x_D))/(y_C^2+x_C^2)-x_D)*(y_H-y_D))/y_D)+1)/2)^2+(y-((x_C*((x_C*(y_C*y_D+x_C*x_D))/(y_C^2+x_C^2)-x_D)*(y_H-y_D))/(y_C*y_D)+y_D)/2)^2-(1-((x_D-(((x_C*(y_C*y_D+x_C*x_D))/(y_C^2+x_C^2)-x_D)*(y_H-y_D))/y_D)+1)/2)^2-(((x_C*((x_C*(y_C*y_D+x_C*x_D))/(y_C^2+x_C^2)-x_D)*(y_H-y_D))/(y_C*y_D)+y_D)/2)^2 = 0$.
γ is given by the equation
$(x-(x_C+x_D)/2)^2+(y-(y_C+y_H)/2)^2-(x_C-(x_C+x_D)/2)^2-(y_C-(y_C+y_H)/2)^2 = 0$.
K has coordinates
$((y_D * (x_D * (x_C - 1)^2 + y_C^2) + y_C * y_H * (y_C * (x_D - 1) - y_D * (x_C - 1)) + y_C * y_D^2 * (x_C - 1))/(y_D * ((x_C - 1)^2 + y_C^2)), 0)$.
L has coordinates
$((x_C * (y_D * (y_C^2 * (x_D * (x_C - 1) + 1) + x_C * x_D * (x_C - 1)^2) + y_C * y_H * (y_C * (x_D - 1) - y_D * (x_C - 1)) + y_C * y_D^2 * (y_C^2 + x_C * (x_C - 1))))/(y_D * (x_C^2 + y_C^2) * ((x_C - 1)^2 + y_C^2)), (y_C * (y_D * (y_C^2 * (x_D * (x_C - 1) + 1) + x_C * x_D * (x_C - 1)^2) + y_C * y_H * (y_C * (x_D - 1) - y_D * (x_C - 1)) + y_C * y_D^2 * (y_C^2 + x_C * (x_C - 1))))/(y_D * (x_C^2 + y_C^2) * ((x_C - 1)^2 + y_C^2)))$.
M has coordinates
$((x_C^2 * x_D * y_D + x_C * y_C * y_D^2 - y_C * y_H * (x_C * y_D - x_D * y_C))/(y_D * (x_C^2 + y_C^2)), 0)$.
N has coordinates
$((y_D * (y_C^4 + x_C * y_C^2 * (x_C + x_D * (x_C - 1)) + x_C^2 * x_D * (x_C - 1)^2) + y_C * y_H * (x_C * y_D - x_D * y_C) * (x_C - 1) + y_C * y_D^2 * (x_C - 1) * (y_C^2 + x_C * (x_C - 1)))/(y_D * (x_C^2 + y_C^2) * ((x_C - 1)^2 + y_C^2)), (y_C * (y_D * (y_C^2 * (x_C * (x_D - 1) + 1) + x_C^2 - x_C^3 + x_C^2 * x_D * (x_C - 1)) + y_C * y_H * (x_C * y_D - x_D * y_C) + y_C * y_D^2 * (y_C^2 + x_C * (x_C - 1))))/(y_D * (x_C^2 + y_C^2) * ((x_C - 1)^2 + y_C^2)))$.
O has coordinates
$((x_C * (x_C * x_D + y_C * y_H))/(x_C^2 + y_C^2), (y_H * y_C^2 + x_C * x_D * y_C)/(x_C^2 + y_C^2))$.
P has coordinates
$((y_C * ((x_C - 1) * y_H + y_C) + (x_C - 1)^2 * x_D)/(y_C^2 + (x_C - 1)^2), (y_C * (y_C * y_H + (x_C - 1) * x_D - x_C + 1))/(y_C^2 + (x_C - 1)^2))$.
The centroid of ABC has coordinates $((x_C + 1)/3, y_C/3)$.
The centroid of ANP has coordinates
$((((y_D * (y_C^4 + x_C * y_C^2 * (x_C + x_D * (x_C - 1)) + x_C^2 * x_D * (x_C - 1)^2) + y_C * y_H * (x_C * y_D - x_D * y_C) * (x_C - 1) + y_C * y_D^2 * (x_C - 1) * (y_C^2 + x_C * (x_C - 1)))/(y_D * (x_C^2 +$

$y_C^2)*((x_C-1)^2+y_C^2)))+((y_C*((x_C-1)*y_H+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)))/3,(((y_C*(y_D*(y_C^2*(x_C*(x_D-1)+1)+x_C^2-x_C^3+x_C^2*x_D*(x_C-1))+y_C*y_H*(x_C*y_D-x_D*y_C)+y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((y_C*(y_C*y_H+(x_C-1)*x_D-x_C+1))/(y_C^2+(x_C-1)^2)))/3).$
The centroid of BLO has coordinates
$((1+((x_C*(y_D*(y_C^2*(x_D*(x_C-1)+1)+x_C*x_D*(x_C-1)^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((x_C*(x_C*x_D+y_C*y_H))/(x_C^2+y_C^2)))/3,(((y_C*(y_D*(y_C^2*(x_D*(x_C-1)+1)+x_C*x_D*(x_C-1)^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((y_H*y_C^2+x_C*x_D*y_C)/(x_C^2+y_C^2)))/3).$
The centroid of CKM has coordinates
$((x_C+((y_D*(x_D*(x_C-1)^2+y_C^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(x_C-1))/(y_D*((x_C-1)^2+y_C^2)))+((x_C^2*x_D*y_D+x_C*y_C*y_D^2-y_C*y_H*(x_C*y_D-x_D*y_C))/(y_D*(x_C^2+y_C^2))))/3,y_C/3).$
Consider the circle whose equation is given by
$(x-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2+(y-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-((x_C+1)/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-(y_C/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2=0.$
We verify that the centroids of ABC, ANP, BLO, and CKM lie on this circle since we calculate
$((x_C+1)/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*$

$x_C+1)*y_D)))^2+(y_C/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*$
$y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*$
$y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-$
$x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-$
$x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+$
$2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+$
$x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*$
$y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-((x_C+1)/3-(-(2*x_C*y_C^3*y_D*y_H-$
$y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*$
$y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*$
$x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*$
$y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*$
$y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*$
$y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-(y_C/3-((y_C^5*y_D*y_H+4*$
$x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*$
$y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-$
$2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+$
$y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*$
$y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*$
$y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+$
$3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2=0,$
$((((y_D*(y_C^4+x_C*y_C^2*(x_C+x_D*(x_C-1))+x_C^2*x_D*(x_C-1)^2)+y_C*y_H*(x_C*y_D-$
$x_D*y_C)*(x_C-1)+y_C*y_D^2*(x_C-1)*(y_C^2+x_C*(x_C-1)))/(y_D*(x_C^2+y_C^2)*((x_C-$
$1)^2+y_C^2)))+((y_C*((x_C-1)*y_H+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2)))/3-(-(2*$
$x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*$
$y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+$
$x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*$
$x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*$
$y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*$
$y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2+((((y_C*(y_D*$
$(y_C^2*(x_C*(x_D-1)+1)+x_C^2-x_C^3+x_C^2*x_D*(x_C-1))+y_C*y_H*(x_C*y_D-x_D*y_C)+$
$y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((y_C*(y_C*y_H+$
$(x_C-1)*x_D-x_C+1))/(y_C^2+(x_C-1)^2)))/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*$
$x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*$
$y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+$
$3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+$
$2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-$
$2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-$
$x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+$
$x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-((x_C+1)/3-(-(2*x_C*y_C^3*$
$y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*$
$y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-$
$2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-$
$2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*$
$y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*$
$y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-(y_C/3-((y_C^5*y_D*y_H+4*$
$x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*$
$y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-$
$2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+$

$y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2=0,$
$((1+((x_C*(y_D*(y_C^2*(x_D*(x_C-1)+1)+x_C*x_D*(x_C-1)^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((x_C*(x_C*x_D+y_C*y_H))/(x_C^2+y_C^2)))/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2+((((y_C*(y_D*(y_C^2*(x_D*(x_C-1)+1)+x_C*x_D*(x_C-1)^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(y_C^2+x_C*(x_C-1))))/(y_D*(x_C^2+y_C^2)*((x_C-1)^2+y_C^2)))+((y_H*y_C^2+x_C*x_D*y_C)/(x_C^2+y_C^2)))/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-((x_C+1)/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-(y_C/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2=0,$

and

$((x_C+((y_D*(x_D*(x_C-1)^2+y_C^2)+y_C*y_H*(y_C*(x_D-1)-y_D*(x_C-1))+y_C*y_D^2*(x_C-1))/(y_D*((x_C-1)^2+y_C^2)))+((x_C^2*x_D*y_D+x_C*y_C*y_D^2-y_C*y_H*(x_C*y_D-x_D*y_C))/(y_D*(x_C^2+y_C^2))))/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2+(y_C/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*$

$y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-((x_C+1)/3-(-(2*x_C*y_C^3*y_D*y_H-y_C^3*y_D*y_H+2*x_C^3*y_C*y_D*y_H-3*x_C^2*y_C*y_D*y_H+x_C*y_C*y_D*y_H-2*x_D*y_C^4*y_H+y_C^4*y_H-2*x_C^2*x_D*y_C^2*y_H+2*x_C*x_D*y_C^2*y_H-x_D*y_C^2*y_H+x_C^2*y_C^2*y_H-2*x_C*y_C^3*y_D^2+y_C^3*y_D^2-2*x_C^3*y_C*y_D^2+3*x_C^2*y_C*y_D^2-x_C*y_C*y_D^2-2*x_C*y_C^4*y_D-2*y_C^4*y_D-2*x_C^2*x_D*y_C^2*y_D+2*x_C*x_D*y_C^2*y_D-x_D*y_C^2*y_D-4*x_C^3*y_C^2*y_D+x_C^2*y_C^2*y_D-y_C^2*y_D-2*x_C^4*x_D*y_D+4*x_C^3*x_D*y_D-2*x_C^2*x_D*y_D-2*x_C^5*y_D+3*x_C^4*y_D-x_C^2*y_D)/(6*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2-(y_C/3-((y_C^5*y_D*y_H+4*x_C^2*y_C^3*y_D*y_H-4*x_C*y_C^3*y_D*y_H+2*y_C^3*y_D*y_H+3*x_C^4*y_C*y_D*y_H-6*x_C^3*y_C*y_D*y_H+3*x_C^2*y_C*y_D*y_H-2*x_C*x_D*y_C^4*y_H+x_D*y_C^4*y_H+x_C*y_C^4*y_H-y_C^4*y_H-2*x_C^3*x_D*y_C^2*y_H+3*x_C^2*x_D*y_C^2*y_H-x_C*x_D*y_C^2*y_H+x_C^3*y_C^2*y_H-x_C^2*y_C^2*y_H+y_C^5*y_D^2-x_C^4*y_C*y_D^2+2*x_C^3*y_C*y_D^2-x_C^2*y_C*y_D^2+y_C^6*y_D+2*x_C*x_D*y_C^4*y_D-x_D*y_C^4*y_D+x_C^2*y_C^4*y_D-2*x_C*y_C^4*y_D+2*y_C^4*y_D+2*x_C^3*x_D*y_C^2*y_D-3*x_C^2*x_D*y_C^2*y_D+x_C*x_D*y_C^2*y_D-x_C^4*y_C^2*y_D+x_C^3*y_C^2*y_D-x_C^2*y_C^2*y_D+x_C*y_C^2*y_D-x_C^6*y_D+3*x_C^5*y_D-3*x_C^4*y_D+x_C^3*y_D)/(6*y_C*(y_C^2+x_C^2)*(y_C^2+x_C^2-2*x_C+1)*y_D)))^2=0$
as desired. □

This particular observation has been recorded online as Euclid 8023.

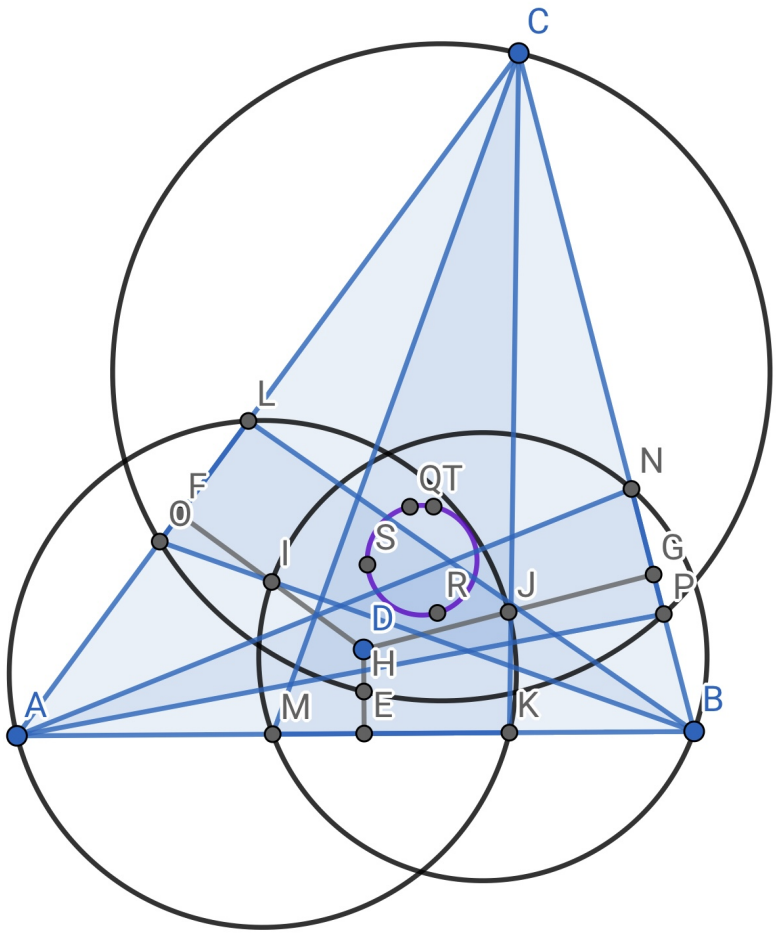

Figure 1. Illustration.

References

[1] Weisstein, Eric W. "Pedal Triangle." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/PedalTriangle.html