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A Line Associated With a Conic Section Passing Through the Sides of a Triangle

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1. Introduction

Pascal's theorem [1] is one of the most fundamental theorems about lines associated with a hexagon whose vertices lie on an ellipse. The result in [2] is another of these sorts of results. Here, we introduce another result about a line that can be found from the same type of hexagon.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D and E be points on line AB, F and G be points on line AC, and H and I be points on line BC such that D, E, F, G, H, and I lie on a conic section.
Let lines AI and BF intersect at J.
Let lines BF and CD intersect at K.
Let lines AH and CD intersect at L.
Let lines AH and BG intersect at M.
Let lines BG and CE intersect at N.
Let lines AI and CE intersect at O.
Then,
1. Lines AN, BL, and CJ are concurrent at P,
2. Lines AK, BO, and CM are concurrent at Q,
3. Lines JM, KN, and LO are concurrent at R, and
4. P, Q, and R are collinear.

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Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates $(x_D, 0)$.
E has coordinates $(x_E, 0)$.
F has coordinates $(x_F, x_F * y_C/x_C)$.
G has coordinates $(x_G, x_G * y_C/x_C)$.
H has coordinates $(x_H, y_C * (x_H - 1)/(x_C - 1))$.
I has coordinates
$(-((((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E) * x_H + ((((1 - x_C^2) * x_D + x_C^2) * x_E + x_C^2 * x_D - x_C^2) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E)/((((x_E + x_D - 1) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E) * x_H + ((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E), -((x_D - 1) * (x_E - 1) * x_F * x_G * (x_H - x_C) * y_C)/((((x_E + x_D - 1) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E) * x_H + ((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E))$.
Line AH is given by the equation
$y = x * (y_C * (x_H - 1)/(x_C - 1))/x_H$.
Line AI is given by the equation
$y = x * (-((x_D - 1) * (x_E - 1) * x_F * x_G * (x_H - x_C) * y_C)/((((x_E + x_D - 1) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E) * x_H + ((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E))/(-((((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E) * x_H + ((((1 - x_C^2) * x_D + x_C^2) * x_E + x_C^2 * x_D - x_C^2) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E)/((((x_E + x_D - 1) * x_F - x_C * x_D * x_E) * x_G - x_C * x_D * x_E * x_F + x_C^2 * x_D * x_E) * x_H + ((((x_C - 1) * x_D - x_C) * x_E - x_C * x_D + x_C) * x_F + x_C * x_D * x_E) * x_G + x_C * x_D * x_E * x_F - x_C^2 * x_D * x_E))$.
Line BF is given by the equation
$y = (x - 1) * (x_F * y_C/x_C)/(x_F - 1)$.
Line BG is given by the equation
$y = (x - 1) * (x_G * y_C/x_C)/(x_G - 1)$.
Line CD is given by the equation
$y = y_C + (x - x_C) * y_C/(x_C - x_D)$.
Line CE is given by the equation
$y = y_C + (x - x_C) * y_C/(x_C - x_E)$.
J has coordinates
$((x_G * (x_F * (x_E * (x_D * (x_C^2 - 1) - x_C^2) - x_C^2 * x_D + x_C^2) + x_C * x_D * x_E) + x_H * (x_G * (x_F * (x_C * x_D - x_C + x_E * (x_C - x_D * (x_C - 1))) - x_C * x_D * x_E) + x_C^2 * x_D * x_E - x_C * x_D * x_E * x_F) - x_C^2 * x_D * x_E + x_C * x_D * x_E * x_F)/(x_H * (x_G * (x_C * x_D - x_C + x_E * (x_C - 2 * x_C * x_D) + x_D * x_E * x_F) + x_C^2 * x_D * x_E - x_C * x_D * x_E * x_F) - x_G * (x_C^2 * x_D - x_C^2 - x_E * (x_D * (x_C^2 + x_C) - x_C^2) + x_D * x_E * x_F) - x_C^2 * x_D * x_E + x_C * x_D * x_E * x_F), (x_F * x_G * y_C * (x_C - x_H) * (x_D - 1) * (x_E - 1))/(x_H * (x_G * (x_C * x_D - x_C + x_E * (x_C - 2 * x_C * x_D) + x_D * x_E * x_F) + x_C^2 * x_D * x_E - x_C * x_D * x_E * x_F) - x_G * (x_C^2 * x_D - x_C^2 - x_E * (x_D * (x_C^2 + x_C) - x_C^2) + x_D * x_E * x_F) - x_C^2 * x_D * x_E + x_C * x_D * x_E * x_F))$.
K has coordinates
$((((x_C + 1) * x_D - x_C) * x_F - x_C * x_D)/(x_D * x_F - x_C), (x_F * (y_C - x_D * y_C))/(x_C - x_D * x_F))$.

L has coordinates
$((x_D * x_H * (x_C - 1))/(x_C - x_D - x_H + x_D * x_H), (x_D * y_C * (x_H - 1))/(x_C - x_D - x_H + x_D * x_H))$.
M has coordinates
$(-(x_G * x_H * (x_C - 1))/(x_C - x_C * x_G - x_C * x_H + x_G * x_H), -(x_G * y_C * (x_H - 1))/(x_C - x_C * x_G - x_C * x_H + x_G * x_H))$.
N has coordinates
$((x_C * x_E + x_C * x_G - x_E * x_G - x_C * x_E * x_G)/(x_C - x_E * x_G), (x_G * (y_C - x_E * y_C))/(x_C - x_E * x_G))$.
O has coordinates
$((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F), (x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F))$.

1. Line AN is given by the equation
$y = x * ((x_G * (y_C - x_E * y_C))/(x_C - x_E * x_G))/((x_C * x_E + x_C * x_G - x_E * x_G - x_C * x_E * x_G)/(x_C - x_E * x_G))$.
Line BL is given by the equation
$y = (x-1) * ((x_D * y_C * (x_H - 1))/(x_C - x_D - x_H + x_D * x_H))/(((x_D * x_H * (x_C - 1))/(x_C - x_D - x_H + x_D * x_H)) - 1)$.
Line CJ is given by the equation
$y = y_C+(x-x_C)*(((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-y_C)/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-x_C)$.
Consider the point P whose coordinates are given by
$(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E), (x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))$.
We verify that P lies on lines AN, BL, and CJ since we calculate
$(-(x_D * (x_C * x_E + x_G * (x_C - x_E * (x_C + 1))) * (x_H - 1))/(x_H * (x_G * (x_E * (3 * x_D - 1) - 2 * x_D + 1) - x_C * x_D * x_E) + x_G * (x_E * (x_C - x_D * (x_C + 2)) - x_C + x_D * (x_C + 1)) + x_C * x_D * x_E)) * ((x_G * (y_C - x_E * y_C))/(x_C - x_E * x_G))/((x_C * x_E + x_C * x_G - x_E * x_G - x_C * x_E * x_G)/(x_C - x_E * x_G)) - ((x_D * x_G * y_C * (x_E - 1) * (x_H - 1))/(x_H * (x_G * (x_E * (3 * x_D - 1) - 2 * x_D + 1) - x_C * x_D * x_E) + x_G * (x_E * (x_C - x_D * (x_C + 2)) - x_C + x_D * (x_C + 1)) + x_C * x_D * x_E)) = 0$,
$(-(x_D * (x_C * x_E + x_G * (x_C - x_E * (x_C + 1))) * (x_H - 1))/(x_H * (x_G * (x_E * (3 * x_D - 1) - 2 * x_D + 1) - x_C * x_D * x_E) + x_G * (x_E * (x_C - x_D * (x_C + 2)) - x_C + x_D * (x_C + 1)) + x_C * x_D * x_E) - 1) * ((x_D * y_C * (x_H - 1))/(x_C - x_D - x_H + x_D * x_H))/(((x_D * x_H * (x_C - 1))/(x_C - x_D - x_H + x_D * x_H)) - 1) - ((x_D * x_G * y_C *$

$(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))=0,$
and
$y_C+(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)-x_C)*(((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-y_C)/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-x_C)-((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))=0$ as desired.

2. Line AK is given by the equation
$y=x*((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))/((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C))$.
Line BO is given by the equation
$y=(x-1)*((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F))/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F))-1)$.
Line CM is given by the equation
$y=y_C+(x-x_C)*((-(x_G*y_C*(x_H-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H))-y_C)/((-(x_G*x_H*(x_C-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H))-x_C)$.
Consider the point Q given by the equation
$((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F), -(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F))$.
We verify that Q lies on lines AK, BO, and CM since we calculate
$((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F))*((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))/((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F))=0,$
$((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)-1)*((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F))/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*$

$(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F))-1)-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F))=0,$
and
$y_C+((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)-x_C)*((-(x_G*y_C*(x_H-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H))-y_C)/((-(x_G*x_H*(x_C-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H))-x_C)-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F))=0$
as desired.

3. Line JM is given by the equation
$y=((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))+(x-((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)))*(((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-(-(x_G*y_C*(x_H-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H)))/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-(-(x_G*x_H*(x_C-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H))))$.
Line KN is given by the equation
$y=((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))+(x-((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C)))*(((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))-((x_G*(y_C-x_E*y_C))/(x_C-x_E*x_G)))/(((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C))-((x_C*x_E+x_C*x_G-x_E*x_G-x_C*x_E*x_G)/(x_C-x_E*x_G)))$.
Line LO is given by the equation
$y=((x_D*y_C*(x_H-1))/(x_C-x_D-x_H+x_D*x_H))+(x-((x_D*x_H*(x_C-1))/(x_C-x_D-x_H+x_D*x_H)))*(((x_D*y_C*(x_H-1))/(x_C-x_D-x_H+x_D*x_H))-((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F)))/(((x_D*x_H*(x_C-1))/(x_C-x_D-x_H+x_D*x_H))-((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F)))$.

Consider the point R whose coordinates are given by
$(-(x_H*(x_G*(x_F*(x_D+x_C*(x_E-1))+x_C*x_D*(x_C-1)-x_C^2*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_F*(x_E*(x_C^2-x_D*(x_C-1)*(x_C+1))+x_C^2*x_D-x_C^2)-x_C*x_D*x_E)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F), -(x_G*y_C*(x_E-1)*(x_C*x_D-x_F*(x_C-x_D*(x_C-1))+x_H*(x_F-x_C*x_D))))/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F))$.
We verify that R lies on lines JM, KN, and LO since we calculate
$((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))+(-(x_H*(x_G*(x_F*(x_D+x_C*(x_E-1))+x_C*x_D*(x_C-1)-x_C^2*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_F*(x_E*(x_C^2-x_D*(x_C-1)*(x_C+1))+x_C^2*x_D-x_C^2)-x_C*x_D*x_E)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F)-((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)))*(((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-(-(x_G*y_C*(x_H-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H)))/(((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_H*(x_G*(x_C*x_D-x_C+x_E*(x_C-2*x_C*x_D)+x_D*x_E*x_F)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_C^2*x_D-x_C^2-x_E*(x_D*(x_C^2+x_C)-x_C^2)+x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F))-(-(x_G*x_H*(x_C-1))/(x_C-x_C*x_G-x_C*x_H+x_G*x_H)))-(-(x_G*y_C*(x_E-1)*(x_C*x_D-x_F*(x_C-x_D*(x_C-1))+x_H*(x_F-x_C*x_D)))/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F))=0$,
$((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))+(-(x_H*(x_G*(x_F*(x_D+x_C*(x_E-1))+x_C*x_D*(x_C-1)-x_C^2*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_F*(x_E*(x_C^2-x_D*(x_C-1)*(x_C+1))+x_C^2*x_D-x_C^2)-x_C*x_D*x_E)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F)-((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C)))*(((x_F*(y_C-x_D*y_C))/(x_C-x_D*x_F))-((x_G*(y_C-x_E*y_C))/(x_C-x_E*x_G)))/(((((x_C+1)*x_D-x_C)*x_F-x_C*x_D)/(x_D*x_F-x_C))-((x_C*x_E+x_C*x_G-x_E*x_G-x_C*x_E*x_G)/(x_C-x_E*x_G)))-(-(x_G*y_C*(x_E-1)*(x_C*x_D-x_F*(x_C-x_D*(x_C-1))+x_H*(x_F-x_C*x_D)))/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F))=0$,
and
$((x_D*y_C*(x_H-1))/(x_C-x_D-x_H+x_D*x_H))+(-(x_H*(x_G*(x_F*(x_D+x_C*(x_E-1))+x_C*x_D*(x_C-1)-x_C^2*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_F*(x_E*(x_C^2-x_D*(x_C-1)*(x_C+1))+x_C^2*x_D-x_C^2)-x_C*x_D*x_E)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*$

$x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F)-((x_D*x_H*(x_C-1))/(x_C-x_D-x_H+x_D*x_H)))*(((x_D*y_C*(x_H-1))/(x_C-x_D-x_H+x_D*x_H))-((x_F*x_G*y_C*(x_C-x_H)*(x_D-1)*(x_E-1))/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F)))/(((x_D*x_H*(x_C-1))/(x_C-x_D-x_H+x_D*x_H))-((x_G*(x_F*(x_E*(x_D*(x_C^2-1)-x_C^2)-x_C^2*x_D+x_C^2)+x_C*x_D*x_E)+x_H*(x_G*(x_F*(x_C*x_D-x_C+x_E*(x_C-x_D*(x_C-1)))-x_C*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_C*x_D-x_F*(x_D*(x_C+1)-x_C+x_E*(x_C-x_C*x_D)))-x_C^2*x_D-x_H*(x_G*(x_C*x_D+x_F*(x_E*(x_D-1)-2*x_D+1))-x_C^2*x_D+x_C*x_D*x_F)+x_C*x_D*x_F)))-(-(x_G*y_C*(x_E-1)*(x_C*x_D-x_F*(x_C-x_D*(x_C-1))+x_H*(x_F-x_C*x_D)))/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F))=0$ as desired.

4. Consider the line whose equation is given by
$y=((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))+(x-(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)))*(((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))/((-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)-((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))-(x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))$.
We verify that P, Q, and R lie on this line since we calculate
$((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))+(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)-(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)))*(((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))/((-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)-((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))-(x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)=0$,
$((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))+((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+$

$x_C*x_D*x_F)-(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)))*(((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))/((-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)) = 0,$

and

$((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))+(-(x_H*(x_G*(x_F*(x_D+x_C*(x_E-1))+x_C*x_D*(x_C-1)-x_C^2*x_D*x_E)+x_C^2*x_D*x_E-x_C*x_D*x_E*x_F)-x_G*(x_F*(x_E*(x_C^2-x_D*(x_C-1)*(x_C+1))+x_C^2*x_D-x_C^2)-x_C*x_D*x_E)-x_C^2*x_D*x_E+x_C*x_D*x_E*x_F)/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F)-(-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E)))*(((x_D*x_G*y_C*(x_E-1)*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-(-(x_F*x_G*y_C*(x_C-x_H)*(x_D-1))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))/((-(x_D*(x_C*x_E+x_G*(x_C-x_E*(x_C+1)))*(x_H-1))/(x_H*(x_G*(x_E*(3*x_D-1)-2*x_D+1)-x_C*x_D*x_E)+x_G*(x_E*(x_C-x_D*(x_C+2))-x_C+x_D*(x_C+1))+x_C*x_D*x_E))-((x_G*(x_C*x_D+x_F*(x_C-x_D*(x_C+1)))*(x_C-x_H))/(x_H*(x_G*(x_F*(3*x_D-1)-2*x_C*x_D)+x_C^2*x_D-x_C*x_D*x_F)+x_G*(x_D*(x_C^2+x_C)+x_F*(x_C-x_D*(2*x_C+1)))-x_C^2*x_D+x_C*x_D*x_F)))-(-(x_G*y_C*(x_E-1)*(x_C*x_D-x_F*(x_C-x_D*(x_C-1))+x_H*(x_F-x_C*x_D)))/(x_G*(x_D*x_F*(x_C-x_E*(x_C-1))-x_C^2+x_C*x_E*(x_C-x_D))+x_C^2*x_D+x_H*(x_G*(x_C-x_D*x_F+x_C*x_E*(x_D-1))-x_C^2*x_D+x_C*x_D*x_F)-x_C*x_D*x_F)) = 0$

as desired. □

References

[1] Weisstein, Eric W. "Pascal's Theorem." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/PascalsTheorem.html

[2] Warren, B.L., "Generalization of the Perspectrix Formed by a Triangle and a Cevian Triangle", *International Journal of Computer Discovered Mathematics*, Under Review.

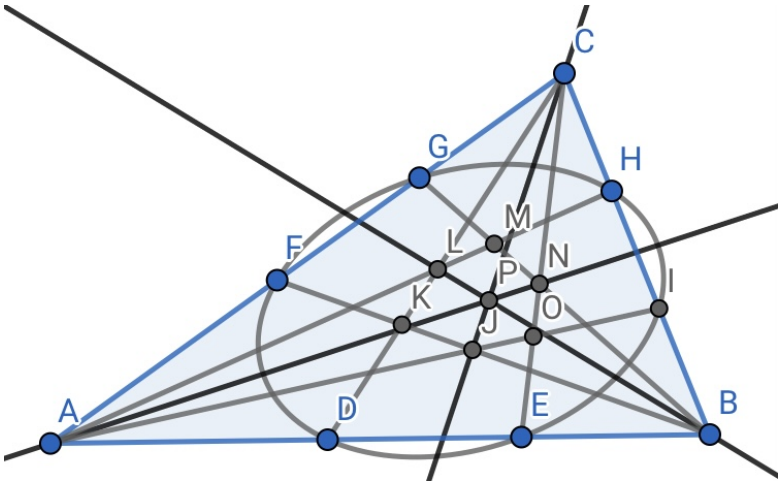

Figure 1. Illustration.

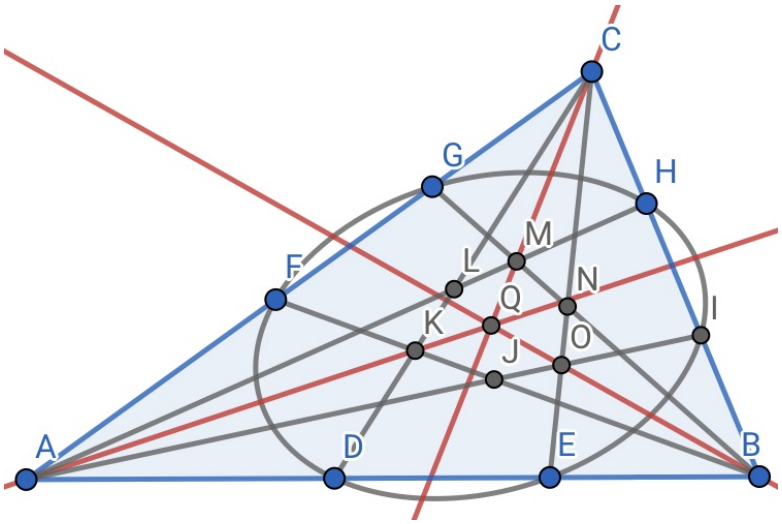

Figure 2. Illustration.

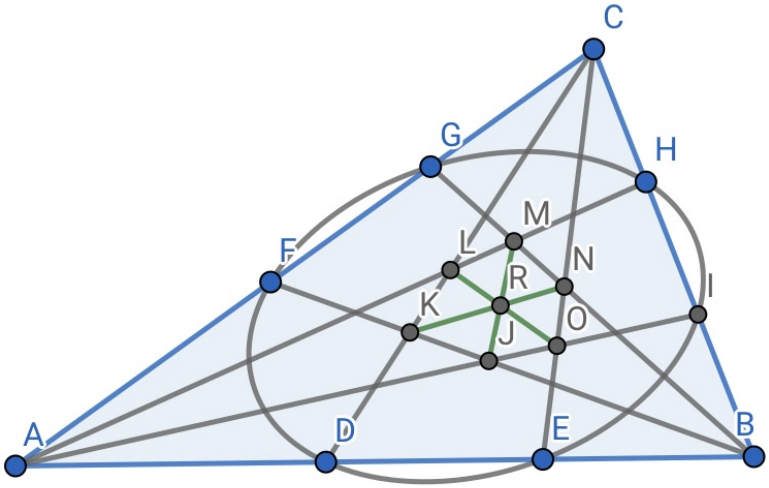

Figure 3. Illustration.

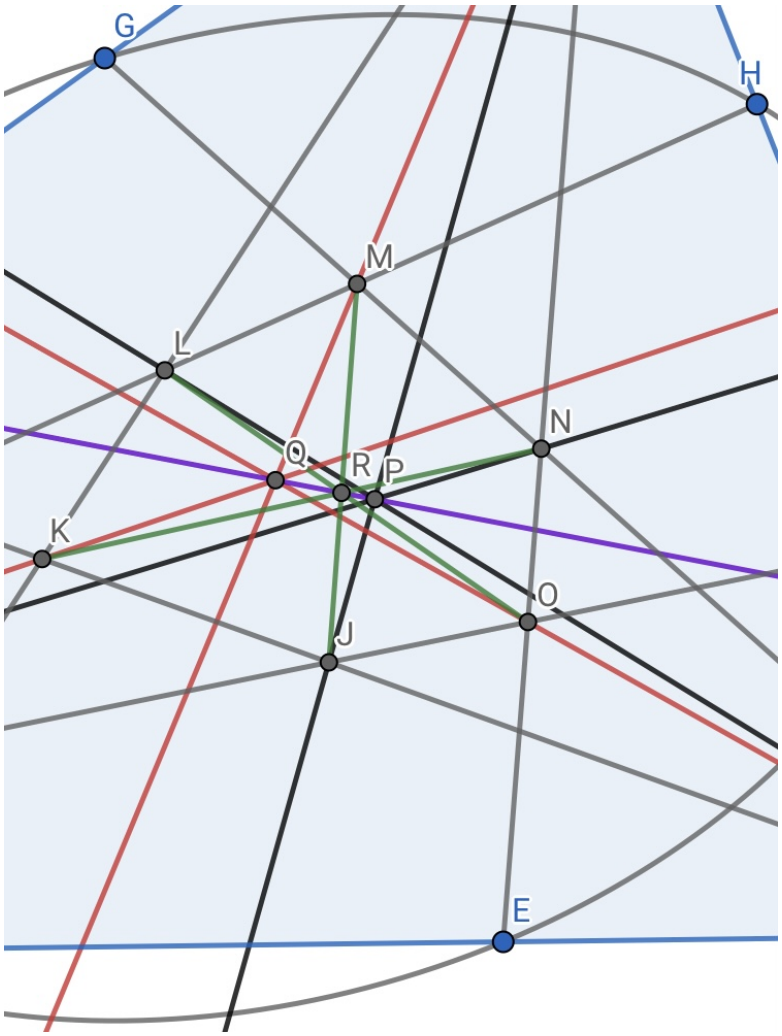

Figure 4. Illustration.