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The Centroid Lies on Another Circle

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1. Introduction

If one forms the pedal and cevian triangles of the centroid of a triangle, then the centroids of the three triangles which comprise a vertex of the other three triangles are concyclic with the centroid about $X(15699)$. We present a proof of this.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D, E, and F be the midpoints of AB, AC, and BC respectively.
Let G be the centroid of ABC.
Let H, I, and J be the vertices of the pedal triangle of G on lines AB, AC, and BC respectively.
Let K, L, and M be the centroids of AFJ, BEI, and CDH respectively.
Then G, K, L, and M are concyclic.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates $(1/2, 0)$.
E has coordinates $(x_C/2, y_C/2)$.
F has coordinates $((x_C + 1)/2, y_C/2)$.
G has coordinates $((x_C + 1)/3, y_C/3)$.
H has coordinates $((x_C + 1)/3, 0)$.
I has coordinates

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$((x_C*(y_C^2+x_C*(x_C+1)))/(3*(y_C^2+x_C^2)), (y_C*(y_C^2+x_C*(x_C+1)))/(3*(y_C^2+x_C^2)))$.
J has coordinates
$(((x_C+2)*y_C^2+(x_C-1)^2*(x_C+1))/(3*(y_C^2+(x_C-1)^2)), (y_C*(y_C^2+(x_C-2)*(x_C-1)))/(3*(y_C^2+(x_C-1)^2)))$.
K has coordinates
$(((5*x_C+7)*y_C^2+5*(x_C-1)^2*(x_C+1))/(18*(y_C^2+(x_C-1)^2)), (y_C*(5*y_C^2+(x_C-1)*(5*x_C-7)))/(18*(y_C^2+(x_C-1)^2)))$.
L has coordinates
$(((5*x_C+6)*y_C^2+x_C^2*(5*x_C+8))/(18*(y_C^2+x_C^2)), (y_C*(5*y_C^2+x_C*(5*x_C+2)))/(18*(y_C^2+x_C^2)))$.
M has coordinates
$((8*x_C+5)/18, y_C/3)$.
Consider the circle whose equation is given by
$(x-(14*x_C+11)/36)^2+(y-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2-(((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2)=0$.
We verify that G, K, L, and M lie on this circle since we calculate
$((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2-(((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2)=0$,
$(((5*x_C+7)*y_C^2+5*(x_C-1)^2*(x_C+1))/(18*(y_C^2+(x_C-1)^2))-(14*x_C+11)/36)^2+((y_C*(5*y_C^2+(x_C-1)*(5*x_C-7)))/(18*(y_C^2+(x_C-1)^2))-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2-(((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2)=0$,
$(((5*x_C+6)*y_C^2+x_C^2*(5*x_C+8))/(18*(y_C^2+x_C^2))-(14*x_C+11)/36)^2+((y_C*(5*y_C^2+x_C*(5*x_C+2)))/(18*(y_C^2+x_C^2))-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2-(((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2)=0$,
and
$((8*x_C+5)/18-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2-(((x_C+1)/3-(14*x_C+11)/36)^2+(y_C/3-((11*y_C)/36-((x_C-1)*x_C)/(12*y_C)))^2)=0$ as desired. □

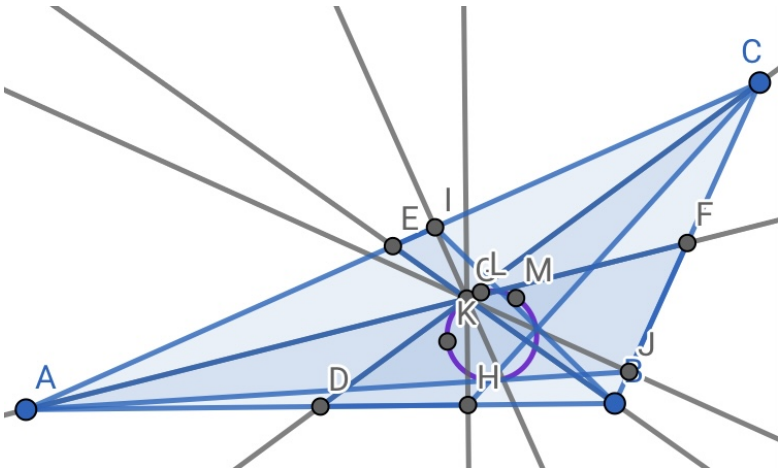

FIGURE 1. Illustration.

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