International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 10, 2025, pp. 41-44
Received January 19, 2025. Published on-line April 11, 2025
web: http://www.journal-1.eu/
©The Author(s) This article is published with open access[1].

Coaxal Circles that Arise from the Pedal and Circumcevian Triangles

Benjamin Lee Warren[a] [2]
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

Mathematics Subject Classification (2010). 51N20.

1. Introduction

We deduce the existence of a set of coaxal circles which pass through the cevian point and the vertices of the pedal and circumcevian triangles.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be a point in the plane.
Let E, F, and G be the vertices of the circumcevian triangle of D adjacent to sides AB, AC, and BC respectively.
Let H, I, and J be the vertices of the pedal triangle of D on sides AB, AC, and BC respectively.
Then circles DEH, DFI, and DGJ are coaxal.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates (x_D, y_D).
E has coordinates
$(((x_C * y_D - x_D * y_C) * (y_C * (y_D - y_C) + (x_C - 1) * (x_D - x_C)))/(y_C * ((y_D - y_C)^2 + (x_D - x_C)^2)), (((x_C - 1) * y_D - (x_D - 1) * y_C) * (x_C * y_D - x_D * y_C))/(y_C * ((y_D - y_C)^2 + (x_D - x_C)^2)))$.
F has coordinates

[1]This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.
[2]Corresponding author

$((y_D*(y_C*(y_D+(x_D-1)*y_C)+(x_C-1)*x_C*(x_D-1)))/(y_C*(y_D^2+(x_D-1)^2)),(y_D*((y_C^2+(x_C-1)*x_C)*y_D-(x_D-1)*y_C))/(y_C*(y_D^2+(x_D-1)^2))).$
G has coordinates
$((x_D*((y_C^2+(x_C-1)*x_C)*y_D+x_D*y_C))/(y_C*(y_D^2+x_D^2)),(y_D*((y_C^2+(x_C-1)*x_C)*y_D+x_D*y_C))/(y_C*(y_D^2+x_D^2))).$
H has coordinates $(x_D,0).$
I has coordinates
$((x_C*(x_C*x_D+y_C*y_D))/(x_C^2+y_C^2),(y_C*(x_C*x_D+y_C*y_D))/(x_C^2+y_C^2)).$
J has coordinates
$((y_C*((x_C-1)*y_D+y_C)+(x_C-1)^2*x_D)/(y_C^2+(x_C-1)^2),(y_C*(y_C*y_D+(x_C-1)*x_D-x_C+1))/(y_C^2+(x_C-1)^2)).$
Circle DEH is given by the equation
$(x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)=0.$
Circle DFI is given by the equation
$(x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2)=0.$
Circle DGJ is given by the equation
$(x-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2)=0.$
The radical line between circles DEH and DFI is given by the equation
$(x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2)=0.$
The radical line between circles DEH and DGJ is given by the equation
$((y_C*y_D+(x_C-1)*x_D)/(y_C*y_D+x_C*(x_D-1)))*((x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2)))=0.$
The radical line between circles DFI and DGJ is given by the equation
$((y_C*y_D+(x_C-1)*x_D)/(x_D-x_C))*((x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-$

$((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2)))=0.$
We then verify that these are all the same line since we calculate
$(x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2))-(((y_C*y_D+(x_C-1)*x_D)/(y_C*y_D+x_C*(x_D-1)))*((x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2))))=0,$
and $(x-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y-y_D/2)^2-((x_D-(((x_C-1)*x_C*y_D+x_D*(x_D-2*x_C+1)*y_C)/(2*(x_D-x_C)*y_C)))^2+(y_D/2)^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2)-(((y_C*y_D+(x_C-1)*x_D)/(x_D-x_C))*((x-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-x_C^2)*y_D+x_C*(x_D-1)*x_D*y_C)/(2*y_C*(y_C*y_D+x_C*(x_D-1)))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+x_C*(x_D+x_C-2)))/(2*(y_C*y_D+x_C*(x_D-1)))))^2)-((x-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2-((x_D-((((x_D+x_C)*y_C^2+x_C^3-2*x_C^2+x_C)*y_D+(x_C-1)*x_D*(x_D+1)*y_C)/(2*y_C*(y_C*y_D+(x_C-1)*x_D))))^2+(y_D-((y_D*(y_C*(y_D+y_C)+(x_C-1)*(x_D+x_C)))/(2*(y_C*y_D+(x_C-1)*x_D))))^2))))=0$
as desired. □

References

[1] Warren, B.L., "The Perspector of the Circumcevian and Antiorthocevian Triangles", *International Journal of Computer Discovered Mathematics*, Vol. 9, pp. 255-258.

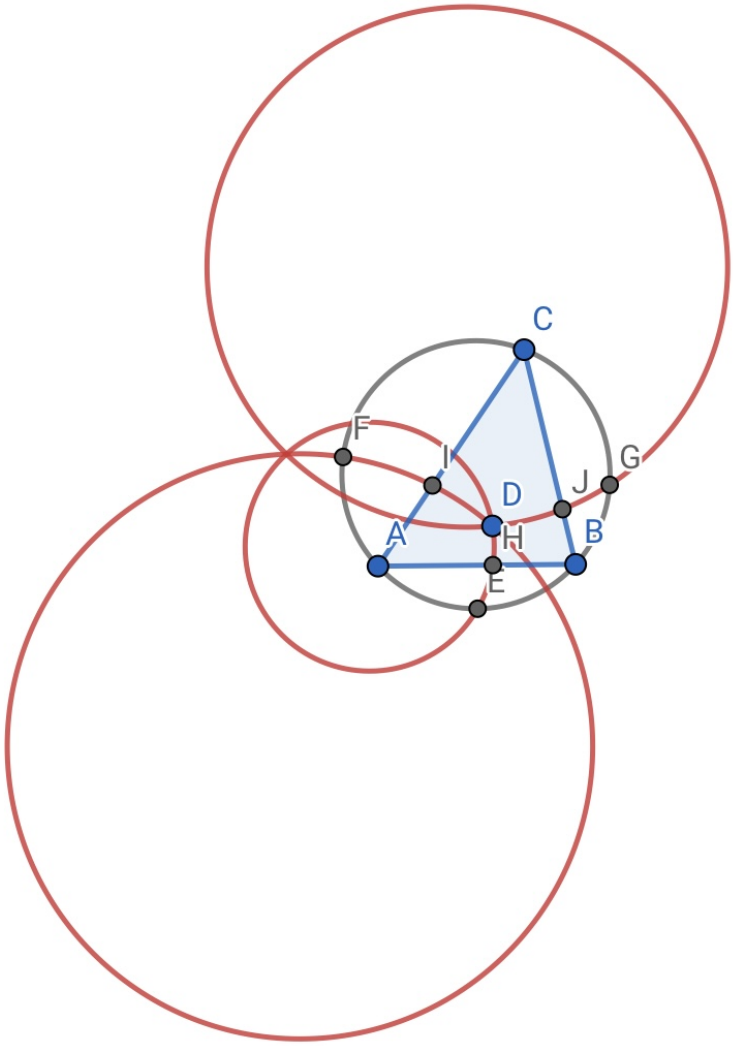

FIGURE 1. Illustration.