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Concurrence of Three Newton Lines

Benjamin Lee Warren[a] [2]
University of Binghamton, Mathematics,
4400 Vestel Pkwy E, Binghamton, NY 13902
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

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1. Introduction

The Newton line of a given quadrilateral is the line that connects the midpoints of the diagonals. It is named so after Newton's theorem, which states that the center of the incircle in a tangential quadrilateral lies on the Newton line [1]. Here, we demonstrate that if a triangle is partitioned by the midpoints into three quadrilaterals, then the Newton lines of those quadrilaterals are concurrent.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be the midpoint of A and B.
Let E be the midpoint of A and C.
Let F be the midpoint of B and C.
Let G be any point in the plane.
Let H be the midpoint of A and G.
Let I be the midpoint of D and E.
Let J be the midpoint of B and G.
Let K be the midpoint of D and F.
Let L be the midpoint of C and G.
Let M be the midpoint of E and F.
Then the Newton lines of the quadrilaterals $\square ADEG$, $\square BDFG$, and $\square CEFG$ are concurrent at point N.

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[2]Corresponding author

Moreover, the midpoint of H and N is I, the midpoint of J and N is K, and the midpoint of L and N is M.

Proof. We work in cartesian coordinates.
A has coordinates given by (x_A, y_A).
B has coordinates given by (x_B, y_B).
C has coordinates given by (x_C, y_C).
D has coordinates given by $(\frac{1}{2}(x_A + x_B), \frac{1}{2}(y_A + y_B))$.
E has coordinates given by $(\frac{1}{2}(x_A + x_C), \frac{1}{2}(y_A + y_C))$.
F has coordinates given by $(\frac{1}{2}(x_B + x_C), \frac{1}{2}(y_B + y_C))$.
G has coordinates given by (x_G, y_G).
H has coordinates given by $(\frac{1}{2}(x_A + x_G), \frac{1}{2}(y_A + y_G))$.
I has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_B + x_C) + x_A), \frac{1}{2}(\frac{1}{2}(y_B + y_C) + y_A))$.
J has coordinates given by $(\frac{1}{2}(x_B + x_G), \frac{1}{2}(y_B + y_G))$.
K has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_A + x_C) + x_B), \frac{1}{2}(\frac{1}{2}(y_A + y_C) + y_B))$.
L has coordinates given by $(\frac{1}{2}(x_C + x_G), \frac{1}{2}(y_C + y_G))$.
M has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_A + x_B) + x_C), \frac{1}{2}(\frac{1}{2}(y_A + y_B) + y_C))$.
Consider the point N whose coordinates are given by $(\frac{1}{2}(x_A+x_B+x_C-x_G), \frac{1}{2}(y_A+y_B + y_C - y_G))$.
We verify that the midpoint of H and N is I since
$\frac{1}{2}(x_H+x_N)-x_I = \frac{1}{2}(\frac{1}{2}(x_A+x_G)+\frac{1}{2}(x_A+x_B+x_C-x_G))-\frac{1}{2}(\frac{1}{2}(x_B+x_C)+x_A) = 0$
and
$\frac{1}{2}(y_H+y_N)-y_I = \frac{1}{2}(\frac{1}{2}(y_A+y_G)+\frac{1}{2}(y_A+y_B+y_C-y_G))-\frac{1}{2}(\frac{1}{2}(y_B+y_C)+y_A) = 0.$
We verify that the midpoint of J and N is K since
$\frac{1}{2}(x_J+x_N)-x_K = \frac{1}{2}(\frac{1}{2}(x_B+x_G)+\frac{1}{2}(x_A+x_B+x_C-x_G))-\frac{1}{2}(\frac{1}{2}(x_A+x_C)+x_B) = 0$
and
$\frac{1}{2}(y_J+y_N)-y_K = \frac{1}{2}(\frac{1}{2}(y_B+y_G)+\frac{1}{2}(y_A+y_B+y_C-y_G))-\frac{1}{2}(\frac{1}{2}(y_A+y_C)+y_B) = 0.$
We verify that the midpoint of L and N is M since
$\frac{1}{2}(x_L+x_N)-x_M = \frac{1}{2}(\frac{1}{2}(x_C+x_G)+\frac{1}{2}(x_A+x_B+x_C-x_G))-\frac{1}{2}(\frac{1}{2}(x_A+x_B)+x_C) = 0$
and
$\frac{1}{2}(y_L+y_N)-y_M = \frac{1}{2}(\frac{1}{2}(y_C+y_G)+\frac{1}{2}(y_A+y_B+y_C-y_G))-\frac{1}{2}(\frac{1}{2}(y_A+y_B)+y_C) = 0.$

□

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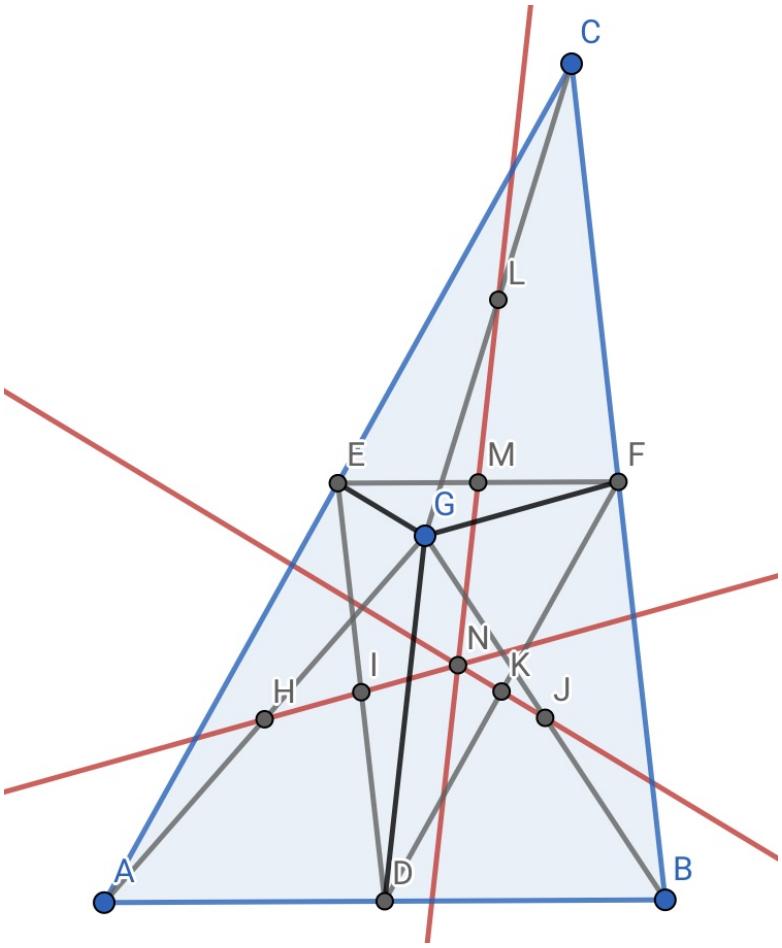

Figure 1. The Newton lines of $\square ADEG$, $\square BDFG$, and $\square CEFG$ are concurrent at point N. The midpoint of H and N is I, the midpoint of J and N is K, and the midpoint of L and N is M.