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Certain Concurrent Circles in a Complete Quadrilateral

Benjamin Lee Warren[a] [2]
University of Binghamton, Mathematics,
4400 Vestel Pkwy E, Binghamton, NY 13902
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

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1. Introduction

The Steiner-Miquel theorem [1] is a famous result about concurrent circles in a complete quadrilateral. Here, we will construct several circles in reference to a complete quadrilateral and will show that they are concurrent.

2. Results

Theorem 2.1. *Let A, B, C, and D be the vertices of a given quadrilateral arranged counterclockwise.*
Let line AD intersect line BC at point E.
Let line AB intersect line CD at point F.
Let G be the midpoint of E and F.
Then circles ABC, AFG, and CEG are concurrent.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates (x_D, y_D).
E has coordinates $((x_D * y_C)/(-x_C * y_D + y_D + x_D * y_C), (y_C * y_D)/(-x_C * y_D + y_D + x_D * y_C))$.
F has coordinates $((x_D * y_C - x_C * y_D)/(y_C - y_D), 0)$.

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[2]Corresponding author

G has coordinates $(((x_D*y_C)/(-x_C*y_D+y_D+x_D*y_C)+(x_D*y_C-x_C*y_D)/(y_C-y_D))/2, (y_C*y_D)/(2*(-x_C*y_D+y_D+x_D*y_C)))$.
Circle ABC is given by the equation
$(x-1/2)^2+(y-(y_C^2+(x_C-1)*x_C)/(2*y_C))^2-(1/2)^2-((y_C^2+(x_C-1)*x_C)/(2*y_C))^2=0$.
Circle AFG is given by the equation
$(x-(x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2+(y-((y_C^2*(y_C-y_D)^2*y_D^2-((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)-x_D*y_C*(y_C-y_D))*((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)+x_D*y_C*(y_C-y_D)))/(4*y_C*(y_C-y_D)^2*y_D*(-x_C*y_D+y_D+x_D*y_C))))^2-((x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2-(((y_C^2*(y_C-y_D)^2*y_D^2-((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)-x_D*y_C*(y_C-y_D))*((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)+x_D*y_C*(y_C-y_D)))/(4*y_C*(y_C-y_D)^2*y_D*(-x_C*y_D+y_D+x_D*y_C))))^2=0$. Circle CEG is given by the equation
$(x+(((2*x_C-1)*y_C^2-x_C^4+4*x_C^3-5*x_C^2+2*x_C)*y_D^4-2*y_C*((x_D+2*x_C-1)*y_C^2-(x_C-1)^2*(2*x_C+1)*x_D+2*(x_C-1)^2*x_C)*y_D^3+y_C^2*((4*x_D+2*x_C-1)*y_C^2-2*(x_C-1)*x_D*((3*x_C+2)*x_D-x_C-2)+2*(x_C-1)^2*x_C)*y_D^2-2*x_D*y_C^3*(y_C^2-2*x_D*(x_C*(x_D+1)-2)+x_C^2-1)*y_D-(x_D-1)*x_D^2*(x_D+3)*y_C^4)/(4*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2+(y-((x_C-1)*((2*x_C^2-4*x_C+3)*y_C^2+(x_C-1)^2*x_C^2)*y_D^4-2*(x_C-1)*y_C*(((3*x_C-4)*x_D+x_C^2-2*x_C+3)*y_C^2+(x_C-1)^2*x_C*(x_D+x_C))*y_D^3+y_C^2*((2*x_D*((3*x_C-4)*x_D+3*x_C^2-8*x_C+6)+3*x_C-3)*y_C^2+6*(x_C-1)^3*x_C*x_D)*y_D^2-2*x_D*y_C^3*((x_D*(x_D+3*x_C-5)-x_C+2)*y_C^2-(x_C-1)^2*x_D*(x_D-3*x_C+1)-(x_C-1)^2*x_C)*y_D+2*(x_D-1)*x_D^2*y_C^6-(x_C-1)*(x_D-1)*x_D^2*(x_D-2*x_C+1)*y_C^4)/(4*y_C*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2-(x_C+(((2*x_C-1)*y_C^2-x_C^4+4*x_C^3-5*x_C^2+2*x_C)*y_D^4-2*y_C*((x_D+2*x_C-1)*y_C^2-(x_C-1)^2*(2*x_C+1)*x_D+2*(x_C-1)^2*x_C)*y_D^3+y_C^2*((4*x_D+2*x_C-1)*y_C^2-2*(x_C-1)*x_D*((3*x_C+2)*x_D-x_C-2)+2*(x_C-1)^2*x_C)*y_D^2-2*x_D*y_C^3*(y_C^2-2*x_D*(x_C*(x_D+1)-2)+x_C^2-1)*y_D-(x_D-1)*x_D^2*(x_D+3)*y_C^4)/(4*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2-(y_C-((x_C-1)*((2*x_C^2-4*x_C+3)*y_C^2+(x_C-1)^2*x_C^2)*y_D^4-2*(x_C-1)*y_C*(((3*x_C-4)*x_D+x_C^2-2*x_C+3)*y_C^2+(x_C-1)^2*x_C*(x_D+x_C))*y_D^3+y_C^2*((2*x_D*((3*x_C-4)*x_D+3*x_C^2-8*x_C+6)+3*x_C-3)*y_C^2+6*(x_C-1)^3*x_C*x_D)*y_D^2-2*x_D*y_C^3*((x_D*(x_D+3*x_C-5)-x_C+2)*y_C^2-(x_C-1)^2*x_D*(x_D-3*x_C+1)-(x_C-1)^2*x_C)*y_D+2*(x_D-1)*x_D^2*y_C^6-(x_C-1)*(x_D-1)*x_D^2*(x_D-2*x_C+1)*y_C^4)/(4*y_C*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2=0$.
Consider the point H whose coordinates are given by
$x_H=-((-2*x_C^4*y_C*y_D^3+x_C^4*y_D^4+4*x_C^3*x_D*y_C^2*y_D^2+4*x_C^3*y_C*y_D^3-2*x_C^3*y_D^4-2*x_C^2*x_D^2*y_C^3*y_D-4*x_C^2*x_D^2*y_C^2*y_D^2-6*x_C^2*x_D*y_C^2*y_D^2-2*x_C^2*y_C^3*y_D^3+2*x_C^2*y_C^2*y_D^4-2*x_C^2*y_C*y_D^3+x_C^2*y_D^4+4*x_C*x_D^3*y_C^3*y_D+2*x_C*x_D^2*y_C^3*y_D+4*x_C*x_D^2*y_C^2*y_D^2+4*x_C*x_D*y_C^4*y_D^2-4*x_C*x_D*y_C^3*y_D^3+2*x_C*x_D*y_C^2*y_D^2+2*x_C*y_C^3*y_D^3-2*x_C*y_C^2*y_D^4-x_D^4*y_C^4-2*x_D^3*y_C^3*y_D-2*x_D^2*y_C^5*y_D+2*x_D^2*y_C^4*y_D^2+x_D^2*y_C^4-2*x_D^2*y_C^3*y_D-2*x_D*y_C^4*y_D^2+2*x_D*y_C^3*y_D^3+y_C^4*y_D^2-2*y_C^3*y_D^3+y_C^2*y_D^4)*(x_C^4*y_D^4-4*x_C^3*x_D*y_C*y_D^3-2*x_C^3*y_C^2*y_D^2+4*x_C^3*y_C*y_D^3-4*x_C^3*y_D^4+6*x_C^2*x_D^2*y_C^2*y_D^2+2*x_C^2*x_D*y_C^3*y_D-4*x_C^2*x_D*y_C^2*y_D^2+8*x_C^2*x_D*y_C*y_D^3+4*x_C^2*y_C^2*y_D^2-8*x_C^2*y_C*y_D^3+5*x_C^2*y_D^4-4*x_C*x_D^3*y_C^3*y_D-6*x_C*x_D^2*y_C^2*y_D^2-2*x_C*x_D*y_C^3*y_D+4*x_C*x_D*y_C^2*y_D^2-4*x_C*x_D*y_C*y_D^3-2*x_C*y_C^4*y_D^2+4*x_C*y_C^3*y_D^3-2*x_C*y_C^2*y_D^4-2*x_C*y_C^2*y_D^2+4*x_C*y_C*y_D^3-2*x_C*y_D^4+x_D^4*y_C^4+2*x_D^3*y_C^3*y_D-x_D^2*y_C^4+2*x_D^2*y_C^3*y_D+2*x_D*y_C^5*y_D-4*x_D*y_C^4*y_D^2+2*x_D*y_C^3*y_D^3+y_C^4*y_D^2-2*y_C^3*y_D^3+y_C^2*y_D^4))/(x_C^8*y_D^8-8*x_C^7*x_D*y_C*y_D^7-4*x_C^7*y_C^2*y_D^6+8*x_C^7*y_C*y_D^7-8*x_C^7*y_D^8+28*x_C^6*x_D^2*y_C^2*y_D^6+20*x_C^6*x_D*y_C^3*$

$y_D^5-40*x_C^6*x_D*y_C^2*y_D^6+48*x_C^6*x_D*y_C*y_D^7+4*x_C^6*y_C^4*y_D^4-16*x_C^6*y_C^3*y_D^5+40*x_C^6*$
$y_C^2*y_D^6-48*x_C^6*y_C*y_D^7+26*x_C^6*y_D^8-56*x_C^5*x_D^3*y_C^3*y_D^5-40*x_C^5*x_D^2*y_C^4*y_D^4+80*$
$x_C^5*x_D^2*y_C^3*y_D^5-124*x_C^5*x_D^2*y_C^2*y_D^6-8*x_C^5*x_D*y_C^5*y_D^3+32*x_C^5*x_D*y_C^4*y_D^4-116*$
$x_C^5*x_D*y_C^3*y_D^5+168*x_C^5*x_D*y_C^2*y_D^6-112*x_C^5*x_D*y_C*y_D^7-4*x_C^5*y_C^4*y_D^6-16*x_C^5*$
$y_C^4*y_D^4+8*x_C^5*y_C^3*y_D^7+64*x_C^5*y_C^3*y_D^5-4*x_C^5*y_C^2*y_D^8-120*x_C^5*y_C^2*y_D^6+112*x_C^5*$
$y_C*y_D^7-44*x_C^5*y_D^8+70*x_C^4*x_D^4*y_C^4*y_D^4+40*x_C^4*x_D^3*y_C^5*y_D^3-80*x_C^4*x_D^3*y_C^4*y_D^4+$
$180*x_C^4*x_D^3*y_C^3*y_D^5+4*x_C^4*x_D^2*y_C^6*y_D^2-16*x_C^4*x_D^2*y_C^5*y_D^3+134*x_C^4*x_D^2*y_C^4*y_D^4-$
$236*x_C^4*x_D^2*y_C^3*y_D^5+204*x_C^4*x_D^2*y_C^2*y_D^6+20*x_C^4*x_D*y_C^5*y_D^5+24*x_C^4*x_D*y_C^5*y_D^3-$
$40*x_C^4*x_D*y_C^4*y_D^6-96*x_C^4*x_D*y_C^4*y_D^4+20*x_C^4*x_D*y_C^3*y_D^7+228*x_C^4*x_D*y_C^3*y_D^5-$
$264*x_C^4*x_D*y_C^2*y_D^6+128*x_C^4*x_D*y_C*y_D^7+8*x_C^4*y_C^6*y_D^4-32*x_C^4*y_C^5*y_D^5+62*x_C^4*$
$y_C^4*y_D^6+24*x_C^4*y_C^4*y_D^4-60*x_C^4*y_C^3*y_D^7-96*x_C^4*y_C^3*y_D^5+22*x_C^4*y_C^2*y_D^8+160*x_C^4*$
$y_C^2*y_D^6-128*x_C^4*y_C*y_D^7+41*x_C^4*y_D^8-56*x_C^3*x_D^5*y_C^5*y_D^3-20*x_C^3*x_D^4*y_C^6*y_D^2+40*$
$x_C^3*x_D^4*y_C^5*y_D^3-160*x_C^3*x_D^4*y_C^4*y_D^4-80*x_C^3*x_D^3*y_C^5*y_D^3+160*x_C^3*x_D^3*y_C^4*y_D^4-$
$200*x_C^3*x_D^3*y_C^3*y_D^5-40*x_C^3*x_D^2*y_C^6*y_D^4-4*x_C^3*x_D^2*y_C^6*y_D^2+80*x_C^3*x_D^2*y_C^5*y_D^5+$
$16*x_C^3*x_D^2*y_C^5*y_D^3-40*x_C^3*x_D^2*y_C^4*y_D^6-128*x_C^3*x_D^2*y_C^4*y_D^4+224*x_C^3*x_D^2*y_C^3*y_D^5-$
$148*x_C^3*x_D^2*y_C^2*y_D^6-16*x_C^3*x_D*y_C^7*y_D^3+64*x_C^3*x_D*y_C^6*y_D^4-152*x_C^3*x_D*y_C^5*y_D^5-$
$24*x_C^3*x_D*y_C^5*y_D^3+176*x_C^3*x_D*y_C^4*y_D^6+96*x_C^3*x_D*y_C^4*y_D^4-72*x_C^3*x_D*y_C^3*y_D^7-$
$188*x_C^3*x_D*y_C^3*y_D^5+184*x_C^3*x_D*y_C^2*y_D^6-72*x_C^3*x_D*y_C*y_D^7-20*x_C^3*y_C^6*y_D^4+88*$
$x_C^3*y_C^5*y_D^5-160*x_C^3*y_C^4*y_D^6-16*x_C^3*y_C^4*y_D^4+136*x_C^3*y_C^3*y_D^7+64*x_C^3*y_C^3*y_D^5-$
$44*x_C^3*y_C^2*y_D^8-100*x_C^3*y_C^2*y_D^6+72*x_C^3*y_C*y_D^7-20*x_C^3*y_D^8+28*x_C^2*x_D^6*y_C^6*y_D^2+$
$4*x_C^2*x_D^5*y_C^7*y_D-8*x_C^2*x_D^5*y_C^6*y_D^2+88*x_C^2*x_D^5*y_C^5*y_D^3+20*x_C^2*x_D^4*y_C^6*y_D^2-40*$
$x_C^2*x_D^4*y_C^5*y_D^3+110*x_C^2*x_D^4*y_C^4*y_D^4+40*x_C^2*x_D^3*y_C^7*y_D^3-4*x_C^2*x_D^3*y_C^7*y_D-80*$
$x_C^2*x_D^3*y_C^6*y_D^4+16*x_C^2*x_D^3*y_C^6*y_D^2+40*x_C^2*x_D^3*y_C^5*y_D^5+24*x_C^2*x_D^3*y_C^5*y_D^3-80*$
$x_C^2*x_D^3*y_C^4*y_D^4+84*x_C^2*x_D^3*y_C^3*y_D^5+8*x_C^2*x_D^2*y_C^8*y_D^2-32*x_C^2*x_D^2*y_C^7*y_D^3+132*$
$x_C^2*x_D^2*y_C^6*y_D^4-4*x_C^2*x_D^2*y_C^6*y_D^2-200*x_C^2*x_D^2*y_C^5*y_D^5+16*x_C^2*x_D^2*y_C^5*y_D^3+92*$
$x_C^2*x_D^2*y_C^4*y_D^6+14*x_C^2*x_D^2*y_C^4*y_D^4-60*x_C^2*x_D^2*y_C^3*y_D^5+40*x_C^2*x_D^2*y_C^2*y_D^6+28*$
$x_C^2*x_D*y_C^7*y_D^3-136*x_C^2*x_D*y_C^6*y_D^4+288*x_C^2*x_D*y_C^5*y_D^5+8*x_C^2*x_D*y_C^5*y_D^3-280*$
$x_C^2*x_D*y_C^4*y_D^6-32*x_C^2*x_D*y_C^4*y_D^4+100*x_C^2*x_D*y_C^3*y_D^7+56*x_C^2*x_D*y_C^3*y_D^5-48*$
$x_C^2*x_D*y_C^2*y_D^6+16*x_C^2*x_D*y_C*y_D^7+4*x_C^2*y_C^8*y_D^4-16*x_C^2*y_C^7*y_D^5+24*x_C^2*y_C^6*y_D^6+$
$20*x_C^2*y_C^6*y_D^4-16*x_C^2*y_C^5*y_D^7-96*x_C^2*y_C^5*y_D^5+4*x_C^2*y_C^4*y_D^8+174*x_C^2*y_C^4*y_D^6+4*$
$x_C^2*y_C^4*y_D^4-140*x_C^2*y_C^3*y_D^7-16*x_C^2*y_C^3*y_D^5+42*x_C^2*y_C^2*y_D^8+24*x_C^2*y_C^2*y_D^6-16*$
$x_C^2*y_C*y_D^7+4*x_C^2*y_D^8-8*x_C*x_D^7*y_C^7*y_D-28*x_C*x_D^6*y_C^6*y_D^2+4*x_C*x_D^5*y_C^7*y_D-$
$8*x_C*x_D^5*y_C^6*y_D^2-32*x_C*x_D^5*y_C^5*y_D^3-20*x_C*x_D^4*y_C^8*y_D^2+40*x_C*x_D^4*y_C^7*y_D^3-$
$20*x_C*x_D^4*y_C^6*y_D^4-20*x_C*x_D^4*y_C^4*y_D^4-56*x_C*x_D^3*y_C^7*y_D^3+4*x_C*x_D^3*y_C^7*y_D+$
$112*x_C*x_D^3*y_C^6*y_D^4-16*x_C*x_D^3*y_C^6*y_D^2-56*x_C*x_D^3*y_C^5*y_D^5+16*x_C*x_D^3*y_C^5*y_D^3-$
$8*x_C*x_D^3*y_C^3*y_D^5-4*x_C*x_D^2*y_C^8*y_D^2+40*x_C*x_D^2*y_C^7*y_D^3-144*x_C*x_D^2*y_C^6*y_D^4+$
$4*x_C*x_D^2*y_C^6*y_D^2+184*x_C*x_D^2*y_C^5*y_D^5-16*x_C*x_D^2*y_C^5*y_D^3-76*x_C*x_D^2*y_C^4*y_D^6+$
$20*x_C*x_D^2*y_C^4*y_D^4-8*x_C*x_D^2*y_C^3*y_D^5-8*x_C*x_D*y_C^9*y_D^3+32*x_C*x_D*y_C^8*y_D^4-48*$
$x_C*x_D*y_C^7*y_D^5-20*x_C*x_D*y_C^7*y_D^3+32*x_C*x_D*y_C^6*y_D^6+112*x_C*x_D*y_C^6*y_D^4-8*$
$x_C*x_D*y_C^5*y_D^7-228*x_C*x_D*y_C^5*y_D^5+200*x_C*x_D*y_C^4*y_D^6-64*x_C*x_D*y_C^3*y_D^7-4*$
$x_C*y_C^8*y_D^4+16*x_C*y_C^7*y_D^5-24*x_C*y_C^6*y_D^6-12*x_C*y_C^6*y_D^4+16*x_C*y_C^5*y_D^7+56*$
$x_C*y_C^5*y_D^5-4*x_C*y_C^4*y_D^8-96*x_C*y_C^4*y_D^6+72*x_C*y_C^3*y_D^7-20*x_C*y_C^2*y_D^8+x_D^8*$
$y_C^8+4*x_D^7*y_C^7*y_D-2*x_D^6*y_C^8+4*x_D^6*y_C^7*y_D+4*x_D^6*y_C^6*y_D^2+4*x_D^5*y_C^9*y_D-8*x_D^5*$
$y_C^8*y_D^2+4*x_D^5*y_C^7*y_D^3-4*x_D^5*y_C^7*y_D+8*x_D^5*y_C^6*y_D^2+14*x_D^4*y_C^8*y_D^2+x_D^4*y_C^8-28*$
$x_D^4*y_C^7*y_D^3-4*x_D^4*y_C^7*y_D+14*x_D^4*y_C^6*y_D^4+4*x_D^4*y_C^6*y_D^2-4*x_D^3*y_C^9*y_D+8*x_D^3*$
$y_C^8*y_D^2+16*x_D^3*y_C^7*y_D^3-40*x_D^3*y_C^6*y_D^4+20*x_D^3*y_C^5*y_D^5+4*x_D^2*y_C^10*y_D^2-16*x_D^2*$
$y_C^9*y_D^3+24*x_D^2*y_C^8*y_D^4+2*x_D^2*y_C^8*y_D^2-16*x_D^2*y_C^7*y_D^5-24*x_D^2*y_C^7*y_D^3+4*x_D^2*y_C^6*$
$y_D^6+66*x_D^2*y_C^6*y_D^4-68*x_D^2*y_C^5*y_D^5+24*x_D^2*y_C^4*y_D^6+4*x_D*y_C^9*y_D^3-16*x_D*y_C^8*$
$y_D^4+24*x_D*y_C^7*y_D^5+8*x_D*y_C^7*y_D^3-16*x_D*y_C^6*y_D^6-40*x_D*y_C^6*y_D^4+4*x_D*y_C^5*$

$y_D^7+72*x_D*y_C^5*y_D^5-56*x_D*y_C^4*y_D^6+16*x_D*y_C^3*y_D^7+y_C^8*y_D^4-4*y_C^7*y_D^5+6*y_C^6*y_D^6+4*y_C^6*y_D^4-4*y_C^5*y_D^7-16*y_C^5*y_D^5+y_C^4*y_D^8+24*y_C^4*y_D^6-16*y_C^3*y_D^7+4*y_C^2*y_D^8)$, and

$y_H = (4*x_C^6*y_C^3*y_D^6-6*x_C^6*y_C^2*y_D^7+2*x_C^6*y_C*y_D^8-16*x_C^5*x_D*y_C^4*y_D^5+20*x_C^5*x_D*y_C^3*y_D^6-4*x_C^5*x_D*y_C^2*y_D^7+4*x_C^5*y_C^4*y_D^5-22*x_C^5*y_C^3*y_D^6+26*x_C^5*y_C^2*y_D^7-8*x_C^5*y_C*y_D^8+24*x_C^4*x_D^2*y_C^5*y_D^4-18*x_C^4*x_D^2*y_C^4*y_D^5-6*x_C^4*x_D^2*y_C^3*y_D^6-12*x_C^4*x_D*y_C^5*y_D^4+66*x_C^4*x_D*y_C^4*y_D^5-66*x_C^4*x_D*y_C^3*y_D^6+12*x_C^4*x_D*y_C^2*y_D^7+4*x_C^4*y_C^5*y_D^6-8*x_C^4*y_C^4*y_D^7-12*x_C^4*y_C^4*y_D^5+4*x_C^4*y_C^3*y_D^8+42*x_C^4*y_C^3*y_D^6-42*x_C^4*y_C^2*y_D^7+12*x_C^4*y_C*y_D^8-16*x_C^3*x_D^3*y_C^6*y_D^3-8*x_C^3*x_D^3*y_C^5*y_D^4+24*x_C^3*x_D^3*y_C^4*y_D^5+12*x_C^3*x_D^2*y_C^6*y_D^3-64*x_C^3*x_D^2*y_C^5*y_D^4+32*x_C^3*x_D^2*y_C^4*y_D^5+20*x_C^3*x_D^2*y_C^3*y_D^6-16*x_C^3*x_D*y_C^6*y_D^5+32*x_C^3*x_D*y_C^5*y_D^6+28*x_C^3*x_D*y_C^5*y_D^4-16*x_C^3*x_D*y_C^4*y_D^7-92*x_C^3*x_D*y_C^4*y_D^5+76*x_C^3*x_D*y_C^3*y_D^6-12*x_C^3*x_D*y_C^2*y_D^7+4*x_C^3*y_C^6*y_D^5-20*x_C^3*y_C^5*y_D^6+28*x_C^3*y_C^4*y_D^7+12*x_C^3*y_C^4*y_D^5-12*x_C^3*y_C^3*y_D^8-34*x_C^3*y_C^3*y_D^6+30*x_C^3*y_C^2*y_D^7-8*x_C^3*y_C*y_D^8+4*x_C^2*x_D^4*y_C^7*y_D^2+22*x_C^2*x_D^4*y_C^6*y_D^3-26*x_C^2*x_D^4*y_C^5*y_D^4-4*x_C^2*x_D^3*y_C^7*y_D^2+16*x_C^2*x_D^3*y_C^6*y_D^3+40*x_C^2*x_D^3*y_C^5*y_D^4-52*x_C^2*x_D^3*y_C^4*y_D^5+24*x_C^2*x_D^2*y_C^7*y_D^4-48*x_C^2*x_D^2*y_C^6*y_D^5-22*x_C^2*x_D^2*y_C^6*y_D^3+24*x_C^2*x_D^2*y_C^5*y_D^6+58*x_C^2*x_D^2*y_C^5*y_D^4-14*x_C^2*x_D^2*y_C^4*y_D^5-22*x_C^2*x_D^2*y_C^3*y_D^6-12*x_C^2*x_D*y_C^7*y_D^4+60*x_C^2*x_D*y_C^6*y_D^5-84*x_C^2*x_D*y_C^5*y_D^6-20*x_C^2*x_D*y_C^5*y_D^4+36*x_C^2*x_D*y_C^4*y_D^7+50*x_C^2*x_D*y_C^4*y_D^5-34*x_C^2*x_D*y_C^3*y_D^6+4*x_C^2*x_D*y_C^2*y_D^7-10*x_C^2*y_C^6*y_D^5+34*x_C^2*y_C^5*y_D^6-38*x_C^2*y_C^4*y_D^7-4*x_C^2*y_C^4*y_D^5+14*x_C^2*y_C^3*y_D^8+10*x_C^2*y_C^3*y_D^6-8*x_C^2*y_C^2*y_D^7+2*x_C^2*y_C*y_D^8-12*x_C*x_D^5*y_C^7*y_D^2+12*x_C*x_D^5*y_C^6*y_D^3+6*x_C*x_D^4*y_C^7*y_D^2-42*x_C*x_D^4*y_C^6*y_D^3+36*x_C*x_D^4*y_C^5*y_D^4-16*x_C*x_D^3*y_C^8*y_D^3+32*x_C*x_D^3*y_C^7*y_D^4+8*x_C*x_D^3*y_C^7*y_D^2-16*x_C*x_D^3*y_C^6*y_D^5-8*x_C*x_D^3*y_C^6*y_D^3-32*x_C*x_D^3*y_C^5*y_D^4+32*x_C*x_D^3*y_C^4*y_D^5+12*x_C*x_D^2*y_C^8*y_D^3-60*x_C*x_D^2*y_C^7*y_D^4-2*x_C*x_D^2*y_C^7*y_D^2+84*x_C*x_D^2*y_C^6*y_D^5+18*x_C*x_D^2*y_C^6*y_D^3-36*x_C*x_D^2*y_C^5*y_D^6-28*x_C*x_D^2*y_C^5*y_D^4+4*x_C*x_D^2*y_C^4*y_D^5+8*x_C*x_D^2*y_C^3*y_D^6+20*x_C*x_D*y_C^7*y_D^4-68*x_C*x_D*y_C^6*y_D^5+76*x_C*x_D*y_C^5*y_D^6+4*x_C*x_D*y_C^5*y_D^4-28*x_C*x_D*y_C^4*y_D^7-8*x_C*x_D*y_C^4*y_D^5+4*x_C*x_D*y_C^3*y_D^6-2*x_C*y_C^7*y_D^4+14*x_C*y_C^6*y_D^5-30*x_C*y_C^5*y_D^6+26*x_C*y_C^4*y_D^7-8*x_C*y_C^3*y_D^8+2*x_D^6*y_C^8*y_D-2*x_D^6*y_C^7*y_D^2-2*x_D^5*y_C^8*y_D+10*x_D^5*y_C^7*y_D^2-8*x_D^5*y_C^6*y_D^3+4*x_D^4*y_C^9*y_D^2-8*x_D^4*y_C^8*y_D^3-2*x_D^4*y_C^8*y_D+4*x_D^4*y_C^7*y_D^4+12*x_D^4*y_C^6*y_D^3-10*x_D^4*y_C^5*y_D^4-4*x_D^3*y_C^9*y_D^2+20*x_D^3*y_C^8*y_D^3+2*x_D^3*y_C^8*y_D-28*x_D^3*y_C^7*y_D^4-10*x_D^3*y_C^7*y_D^2+12*x_D^3*y_C^6*y_D^5+12*x_D^3*y_C^6*y_D^3-4*x_D^3*y_C^4*y_D^5-10*x_D^2*y_C^8*y_D^3+34*x_D^2*y_C^7*y_D^4+2*x_D^2*y_C^7*y_D^2-38*x_D^2*y_C^6*y_D^5-8*x_D^2*y_C^6*y_D^3+14*x_D^2*y_C^5*y_D^6+10*x_D^2*y_C^5*y_D^4-4*x_D^2*y_C^4*y_D^5+2*x_D*y_C^8*y_D^3-14*x_D*y_C^7*y_D^4+30*x_D*y_C^6*y_D^5-26*x_D*y_C^5*y_D^6+8*x_D*y_C^4*y_D^7+2*y_C^7*y_D^4-8*y_C^6*y_D^5+12*y_C^5*y_D^6-8*y_C^4*y_D^7+2*y_C^3*y_D^8)/(x_C^8*y_D^8-8*x_C^7*x_D*y_C*y_D^7-4*x_C^7*y_C^2*y_D^6+8*x_C^7*y_C*y_D^7-8*x_C^7*y_D^8+28*x_C^6*x_D^2*y_C^2*y_D^6+20*x_C^6*x_D*y_C^3*y_D^5-40*x_C^6*x_D*y_C^2*y_D^6+48*x_C^6*x_D*y_C*y_D^7+4*x_C^6*y_C^4*y_D^4-16*x_C^6*y_C^3*y_D^5+40*x_C^6*y_C^2*y_D^6-48*x_C^6*y_C*y_D^7+26*x_C^6*y_D^8-56*x_C^5*x_D^3*y_C^3*y_D^5-40*x_C^5*x_D^2*y_C^4*y_D^4+80*x_C^5*x_D^2*y_C^3*y_D^5-124*x_C^5*x_D^2*y_C^2*y_D^6-8*x_C^5*x_D*y_C^5*y_D^3+32*x_C^5*x_D*y_C^4*y_D^4-116*x_C^5*x_D*y_C^3*y_D^5+168*x_C^5*x_D*y_C^2*y_D^6-112*x_C^5*x_D*y_C*y_D^7-4*x_C^5*y_C^4*y_D^6-16*x_C^5*y_C^4*y_D^4+8*x_C^5*y_C^3*y_D^7+64*x_C^5*y_C^3*y_D^5-4*x_C^5*y_C^2*y_D^8-120*x_C^5*y_C^2*y_D^6+112*x_C^5*y_C*y_D^7-44*x_C^5*y_D^8+70*x_C^4*x_D^4*y_C^4*y_D^4+40*x_C^4*x_D^3*y_C^5*y_D^3-80*x_C^4*x_D^3*y_C^4*y_D^4+180*x_C^4*x_D^3*y_C^3*y_D^5+4*x_C^4*x_D^2*y_C^6*y_D^2-16*x_C^4*x_D^2*y_C^5*y_D^3+134*x_C^4*x_D^2*y_C^4*y_D^4-236*x_C^4*x_D^2*y_C^3*y_D^5+204*x_C^4*x_D^2*y_C^2*y_D^6+20*x_C^4*x_D*y_C^5*y_D^5+24*x_C^4*x_D*y_C^5*y_D^3-40*x_C^4*x_D*y_C^4*y_D^6-96*x_C^4*x_D*y_C^4*y_D^4+20*x_C^4*x_D*y_C^3*y_D^7+228*x_C^4*x_D*y_C^3*y_D^5-264*x_C^4*x_D*y_C^2*y_D^6+128*x_C^4*x_D*y_C*y_D^7+8*x_C^4*y_C^6*y_D^4-32*x_C^4*y_C^5*y_D^5+62*x_C^4*y_C^4*y_D^6+24*x_C^4*y_C^4*y_D^4-60*x_C^4*y_C^3*y_D^7-96*x_C^4*y_C^3*y_D^5+22*x_C^4*y_C^2*y_D^8+160*x_C^4*y_C^2*y_D^6-128*x_C^4*y_C*y_D^7+41*x_C^4*y_D^8-56*x_C^3*x_D^5*y_C^5*y_D^3-20*x_C^3*x_D^4*y_C^6*y_D^2+40*x_C^3*x_D^4*y_C^5*y_D^3-160*x_C^3*x_D^4*y_C^4*y_D^4-80*x_C^3*x_D^3*y_C^5*y_D^3+160*x_C^3*x_D^3*y_C^4*y_D^4-200*x_C^3*x_D^3*y_C^3*y_D^5-40*x_C^3*x_D^2*y_C^6*$

$y_D^4-4*x_C^3*x_D^2*y_C^6*y_D^2+80*x_C^3*x_D^2*y_C^5*y_D^5+16*x_C^3*x_D^2*y_C^5*y_D^3-40*x_C^3*x_D^2*y_C^4*y_D^6-128*x_C^3*x_D^2*y_C^4*y_D^4+224*x_C^3*x_D^2*y_C^3*y_D^5-148*x_C^3*x_D^2*y_C^2*y_D^6-16*x_C^3*x_D*y_C^7*y_D^3+64*x_C^3*x_D*y_C^6*y_D^4-152*x_C^3*x_D*y_C^5*y_D^5-24*x_C^3*x_D*y_C^5*y_D^3+176*x_C^3*x_D*y_C^4*y_D^6+96*x_C^3*x_D*y_C^4*y_D^4-72*x_C^3*x_D*y_C^3*y_D^7-188*x_C^3*x_D*y_C^3*y_D^5+184*x_C^3*x_D*y_C^2*y_D^6-72*x_C^3*x_D*y_C*y_D^7-20*x_C^3*y_C^6*y_D^4+88*x_C^3*y_C^5*y_D^5-160*x_C^3*y_C^4*y_D^6-16*x_C^3*y_C^4*y_D^4+136*x_C^3*y_C^3*y_D^7+64*x_C^3*y_C^3*y_D^5-44*x_C^3*y_C^2*y_D^8-100*x_C^3*y_C^2*y_D^6+72*x_C^3*y_C*y_D^7-20*x_C^3*y_D^8+28*x_C^2*x_D^6*y_C^6*y_D^2+4*x_C^2*x_D^5*y_C^7*y_D-8*x_C^2*x_D^5*y_C^6*y_D^2+88*x_C^2*x_D^5*y_C^5*y_D^3+20*x_C^2*x_D^4*y_C^6*y_D^2-40*x_C^2*x_D^4*y_C^5*y_D^3+110*x_C^2*x_D^4*y_C^4*y_D^4+40*x_C^2*x_D^3*y_C^7*y_D^3-4*x_C^2*x_D^3*y_C^7*y_D-80*x_C^2*x_D^3*y_C^6*y_D^4+16*x_C^2*x_D^3*y_C^6*y_D^2+40*x_C^2*x_D^3*y_C^5*y_D^5+24*x_C^2*x_D^3*y_C^5*y_D^3-80*x_C^2*x_D^3*y_C^4*y_D^4+84*x_C^2*x_D^3*y_C^3*y_D^5+8*x_C^2*x_D^2*y_C^8*y_D^2-32*x_C^2*x_D^2*y_C^7*y_D^3+132*x_C^2*x_D^2*y_C^6*y_D^4-4*x_C^2*x_D^2*y_C^6*y_D^2-200*x_C^2*x_D^2*y_C^5*y_D^5+16*x_C^2*x_D^2*y_C^5*y_D^3+92*x_C^2*x_D^2*y_C^4*y_D^6+14*x_C^2*x_D^2*y_C^4*y_D^4-60*x_C^2*x_D^2*y_C^3*y_D^5+40*x_C^2*x_D^2*y_C^2*y_D^6+28*x_C^2*x_D*y_C^7*y_D^3-136*x_C^2*x_D*y_C^6*y_D^4+288*x_C^2*x_D*y_C^5*y_D^5+8*x_C^2*x_D*y_C^5*y_D^3-280*x_C^2*x_D*y_C^4*y_D^6-32*x_C^2*x_D*y_C^4*y_D^4+100*x_C^2*x_D*y_C^3*y_D^7+56*x_C^2*x_D*y_C^3*y_D^5-48*x_C^2*x_D*y_C^2*y_D^6+16*x_C^2*x_D*y_C*y_D^7+4*x_C^2*y_C^8*y_D^4-16*x_C^2*y_C^7*y_D^5+24*x_C^2*y_C^6*y_D^6+20*x_C^2*y_C^6*y_D^4-16*x_C^2*y_C^5*y_D^7-96*x_C^2*y_C^5*y_D^5+4*x_C^2*y_C^4*y_D^8+174*x_C^2*y_C^4*y_D^6+4*x_C^2*y_C^4*y_D^4-140*x_C^2*y_C^3*y_D^7-16*x_C^2*y_C^3*y_D^5+42*x_C^2*y_C^2*y_D^8+24*x_C^2*y_C^2*y_D^6-16*x_C^2*y_C*y_D^7+4*x_C^2*y_D^8-8*x_C*x_D^7*y_C^7*y_D-28*x_C*x_D^6*y_C^6*y_D^2+4*x_C*x_D^5*y_C^7*y_D-8*x_C*x_D^5*y_C^6*y_D^2-32*x_C*x_D^5*y_C^5*y_D^3-20*x_C*x_D^4*y_C^8*y_D^2+40*x_C*x_D^4*y_C^7*y_D^3-20*x_C*x_D^4*y_C^6*y_D^4-20*x_C*x_D^4*y_C^4*y_D^4-56*x_C*x_D^3*y_C^7*y_D^3+4*x_C*x_D^3*y_C^7*y_D+112*x_C*x_D^3*y_C^6*y_D^4-16*x_C*x_D^3*y_C^6*y_D^2-56*x_C*x_D^3*y_C^5*y_D^5+16*x_C*x_D^3*y_C^5*y_D^3-8*x_C*x_D^3*y_C^3*y_D^5-4*x_C*x_D^2*y_C^8*y_D^2+40*x_C*x_D^2*y_C^7*y_D^3-144*x_C*x_D^2*y_C^6*y_D^4+4*x_C*x_D^2*y_C^6*y_D^2+184*x_C*x_D^2*y_C^5*y_D^5-16*x_C*x_D^2*y_C^5*y_D^3-76*x_C*x_D^2*y_C^4*y_D^6+20*x_C*x_D^2*y_C^4*y_D^4-8*x_C*x_D^2*y_C^3*y_D^5-8*x_C*x_D*y_C^9*y_D^3+32*x_C*x_D*y_C^8*y_D^4-48*x_C*x_D*y_C^7*y_D^5-20*x_C*x_D*y_C^7*y_D^3+32*x_C*x_D*y_C^6*y_D^6+112*x_C*x_D*y_C^6*y_D^4-8*x_C*x_D*y_C^5*y_D^7-228*x_C*x_D*y_C^5*y_D^5+200*x_C*x_D*y_C^4*y_D^6-64*x_C*x_D*y_C^3*y_D^7-4*x_C*y_C^8*y_D^4+16*x_C*y_C^7*y_D^5-24*x_C*y_C^6*y_D^6-12*x_C*y_C^6*y_D^4+16*x_C*y_C^5*y_D^7+56*x_C*y_C^5*y_D^5-4*x_C*y_C^4*y_D^8-96*x_C*y_C^4*y_D^6+72*x_C*y_C^3*y_D^7-20*x_C*y_C^2*y_D^8+x_D^8*y_C^8+4*x_D^7*y_C^7*y_D-2*x_D^6*y_C^8+4*x_D^6*y_C^7*y_D+4*x_D^6*y_C^6*y_D^2+4*x_D^5*y_C^9*y_D-8*x_D^5*y_C^8*y_D^2+4*x_D^5*y_C^7*y_D^3-4*x_D^5*y_C^7*y_D+8*x_D^5*y_C^6*y_D^2+14*x_D^4*y_C^8*y_D^2+x_D^4*y_C^8-28*x_D^4*y_C^7*y_D^3-4*x_D^4*y_C^7*y_D+14*x_D^4*y_C^6*y_D^4+4*x_D^4*y_C^6*y_D^2-4*x_D^3*y_C^9*y_D+8*x_D^3*y_C^8*y_D^2+16*x_D^3*y_C^7*y_D^3-40*x_D^3*y_C^6*y_D^4+20*x_D^3*y_C^5*y_D^5+4*x_D^2*y_C^10*y_D^2-16*x_D^2*y_C^9*y_D^3+24*x_D^2*y_C^8*y_D^4+2*x_D^2*y_C^8*y_D^2-16*x_D^2*y_C^7*y_D^5-24*x_D^2*y_C^7*y_D^3+4*x_D^2*y_C^6*y_D^6+66*x_D^2*y_C^6*y_D^4-68*x_D^2*y_C^5*y_D^5+24*x_D^2*y_C^4*y_D^6+4*x_D*y_C^9*y_D^3-16*x_D*y_C^8*y_D^4+24*x_D*y_C^7*y_D^5+8*x_D*y_C^7*y_D^3-16*x_D*y_C^6*y_D^6-40*x_D*y_C^6*y_D^4+4*x_D*y_C^5*y_D^7+72*x_D*y_C^5*y_D^5-56*x_D*y_C^4*y_D^6+16*x_D*y_C^3*y_D^7+y_C^8*y_D^4-4*y_C^7*y_D^5+6*y_C^6*y_D^6+4*y_C^6*y_D^4-4*y_C^5*y_D^7-16*y_C^5*y_D^5+y_C^4*y_D^8+24*y_C^4*y_D^6-16*y_C^3*y_D^7+4*y_C^2*y_D^8)$.

One can then verify that H lies on all three circles since we substitute the above values and calculate that

$(x_H-1/2)^2+(y_H-(y_C^2+(x_C-1)*x_C)/(2*y_C))^2-(1/2)^2-((y_C^2+(x_C-1)*x_C)/(2*y_C))^2=0,$

$(x_H-(x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2+(y_H-((y_C^2*(y_C-y_D)^2*y_D^2-((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)-x_D*y_C*(y_C-y_D))*((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)+x_D*y_C*(y_C-y_D)))/(4*y_C*(y_C-y_D)^2*y_D*(-x_C*y_D+y_D+x_D*y_C))))^2-((x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2-(((y_C^2*(y_C-y_D)^2*y_D^2-((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)-x_D*y_C*(y_C-y_D))*((x_D*y_C-x_C*y_D)*(-x_C*y_D+y_D+x_D*y_C)+x_D*y_C*(y_C-y_D)))/(4*y_C*(y_C-y_D)^2*y_D*(-x_C*y_D+y_D+x_D*y_C))))^2=0,$

and

$(x_H+(((2*x_C-1)*y_C^2-x_C^4+4*x_C^3-5*x_C^2+2*x_C)*y_D^4-2*y_C*((x_D+2*x_C-1)*y_C^2-(x_C-1)^2*(2*x_C+1)*x_D+2*(x_C-1)^2*x_C)*y_D^3+y_C^2*((4*x_D+2*x_C-1)*y_C^2-2*(x_C-1)*x_D*((3*x_C+2)*x_D-x_C-2)+2*(x_C-1)^2*x_C)*y_D^2-2*x_D*y_C^3*(y_C^2-2*x_D*(x_C*(x_D+1)-2)+x_C^2-1)*y_D-(x_D-1)*x_D^2*(x_D+3)*y_C^4)/(4*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2+(y_H-((x_C-1)*((2*x_C^2-4*x_C+3)*y_C^2+(x_C-1)^2*x_C^2)*y_D^4-2*(x_C-1)*y_C*(((3*x_C-4)*x_D+x_C^2-2*x_C+3)*y_C^2+(x_C-1)^2*x_C*(x_D+x_C))*y_D^3+y_C^2*((2*x_D*((3*x_C-4)*x_D+3*x_C^2-8*x_C+6)+3*x_C-3)*y_C^2+6*(x_C-1)^3*x_C*x_D)*y_D^2-2*x_D*y_C^3*((x_D*(x_D+3*x_C-5)-x_C+2)*y_C^2-(x_C-1)^2*x_D*(x_D-3*x_C+1)-(x_C-1)^2*x_C)*y_D+2*(x_D-1)*x_D^2*y_C^6-(x_C-1)*(x_D-1)*x_D^2*(x_D-2*x_C+1)*y_C^4)/(4*y_C*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2-(x_C+(((2*x_C-1)*y_C^2-x_C^4+4*x_C^3-5*x_C^2+2*x_C)*y_D^4-2*y_C*((x_D+2*x_C-1)*y_C^2-(x_C-1)^2*(2*x_C+1)*x_D+2*(x_C-1)^2*x_C)*y_D^3+y_C^2*((4*x_D+2*x_C-1)*y_C^2-2*(x_C-1)*x_D*((3*x_C+2)*x_D-x_C-2)+2*(x_C-1)^2*x_C)*y_D^2-2*x_D*y_C^3*(y_C^2-2*x_D*(x_C*(x_D+1)-2)+x_C^2-1)*y_D-(x_D-1)*x_D^2*(x_D+3)*y_C^4)/(4*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2-(y_C-((x_C-1)*((2*x_C^2-4*x_C+3)*y_C^2+(x_C-1)^2*x_C^2)*y_D^4-2*(x_C-1)*y_C*(((3*x_C-4)*x_D+x_C^2-2*x_C+3)*y_C^2+(x_C-1)^2*x_C*(x_D+x_C))*y_D^3+y_C^2*((2*x_D*((3*x_C-4)*x_D+3*x_C^2-8*x_C+6)+3*x_C-3)*y_C^2+6*(x_C-1)^3*x_C*x_D)*y_D^2-2*x_D*y_C^3*((x_D*(x_D+3*x_C-5)-x_C+2)*y_C^2-(x_C-1)^2*x_D*(x_D-3*x_C+1)-(x_C-1)^2*x_C)*y_D+2*(x_D-1)*x_D^2*y_C^6-(x_C-1)*(x_D-1)*x_D^2*(x_D-2*x_C+1)*y_C^4)/(4*y_C*(y_D-y_C)*((x_C-1)*y_D-(x_D-1)*y_C)*((x_C-1)*y_D-x_D*y_C)^2))^2=0$
as desired. □

Theorem 2.2. *Let A, B, C, and D be the vertices of a given quadrilateral arranged counterclockwise.*
Let line AD intersect line BC at point E.
Let line AB intersect line CD at point F.
Let G be the midpoint of A and C.
Then circles BEF, AFG, and CEG are concurrent.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0,0)$.
B has coordinates $(1,0)$.
C has coordinates (x_C, y_C).
D has coordinates (x_D, y_D).
E has coordinates $((x_D*y_C)/(-x_C*y_D+y_D+x_D*y_C),(y_C*y_D)/(-x_C*y_D+y_D+x_D*y_C))$.
F has coordinates $((x_D*y_C-x_C*y_D)/(y_C-y_D),0)$.
G has coordinates $(x_C/2, y_C/2)$.
Circle BEF is given by the equation
$(x-((x_D*y_C-x_C*y_D)/(y_C-y_D)+1)/2)^2+(y+((y_C^2+(x_C-1)^2*x_C)*y_D^2-y_C*(y_C^2+2*(x_C-1)^2*x_D)*y_D+(x_C-1)*(x_D-1)*x_D*y_C^2)/(2*y_C*(y_D-y_C)*((x_C-1)*y_D-x_D*y_C)))^2-(1-((x_D*y_C-x_C*y_D)/(y_C-y_D)+1)/2)^2-(((y_C^2+(x_C-1)^2*x_C)*y_D^2-y_C*(y_C^2+2*(x_C-1)^2*x_D)*y_D+(x_C-1)*(x_D-1)*x_D*y_C^2)/(2*y_C*(y_D-y_C)*((x_C-1)*y_D-x_D*y_C)))^2=0.$
Circle AFG is given by the equation $(x-(x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2+(y-(y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2-((x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2-((y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2=0.$
Circle CEG is given by the equation $(x-(((x_C+1)*y_C^2+x_C^3+x_C^2-2*x_C)*$

$y_D - x_D * y_C^3 + (-x_C^2 - 2) * x_D * y_C)/(4 * ((x_C - 1) * y_D - x_D * y_C)))^2 + (y + (((x_C^2 - 2 * x_C + 3) * y_C^2 + x_C^4 - 2 * x_C^3 + x_C^2) * y_D - (x_C - 3) * x_D * y_C^3 - (x_C - 2) * (x_C - 1) * x_C * x_D * y_C)/(4 * y_C * ((x_C - 1) * y_D - x_D * y_C)))^2 - (x_C - (((x_C + 1) * y_C^2 + x_C^3 + x_C^2 - 2 * x_C) * y_D - x_D * y_C^3 + (-x_C^2 - 2) * x_D * y_C)/(4 * ((x_C - 1) * y_D - x_D * y_C)))^2 - (y_C + (((x_C^2 - 2 * x_C + 3) * y_C^2 + x_C^4 - 2 * x_C^3 + x_C^2) * y_D - (x_C - 3) * x_D * y_C^3 - (x_C - 2) * (x_C - 1) * x_C * x_D * y_C)/(4 * y_C * ((x_C - 1) * y_D - x_D * y_C)))^2 = 0.$

Consider the point H whose coordinates are given by

$x_H = (((x_C^2-1)*y_C^4+(2*x_C-2*x_C^2)*y_C^2-x_C^6+4*x_C^5-5*x_C^4+2*x_C^3)*y_D^4+((-2*x_C*x_D-2*x_C^2+2)*y_C^5+((2*x_C^3-2*x_C^2+2*x_C)*x_D-2*x_C^4+2*x_C^3+2*x_C^2-2*x_C)*y_C^3+((4*x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)$

and

$y_H = -(2*y_C*(((x_C^3-x_C^2+x_C-1)*y_C^2+x_C^5-3*x_C^4+3*x_C^3-x_C^2)*y_D^4+(((-3*x_C^2+2*x_C-1)*x_D-x_C^3+2*x_C^2-3*x_C+2)*y_C^3+((-3*x_C^4+10*x_C^3-11*x_C^2+4*x_C)*x_D-x_C^5+2*x_C^4-x_C^3)*y_C)*y_D^3+(((3*x_C-1)*x_D^2+(3*x_C^2-4*x_C+3)*x_D-x_C^2+2*x_C-1)*y_C^4+((3*x_C^3-13*x_C^2+14*x_C-4)*x_D^2+(3*x_C^4-4*x_C^3+x_C^2)*x_D-x_C^4+2*x_C^3-x_C^2)*y_C^2)*y_D^2+((-x_D^3+(2-3*x_C)*x_D^2+(2*x_C-2)*x_D)*y_C^5+((-x_C^2+8*x_C-6)*x_D^3+(-3*x_C^3+2*x_C^2-2*x_C+2)*x_D^2+(2*x_C^3-2*x_C^2)*x_D)*y_C^3)*y_D+(x_D^3-x_D^2)*y_C^6+(-2*x_D^4+(x_C^2+2)*x_D^3-x_C^2*x_D^2)*y_C^4))/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)$

We verify that H lies on all three circles since we calculate

$((((x_C^2-1)*y_C^4+(2*x_C-2*x_C^2)*y_C^2-x_C^6+4*x_C^5-5*x_C^4+2*x_C^3)*y_D^4+((-2*x_C*x_D-2*x_C^2+2)*y_C^5+((2*x_C^3-2*x_C^2+2*x_C)*x_D-2*x_C^4+2*x_C^3+2*x_C^2-2*x_C)*y_C^3+((4*$

$x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)-((x_D*y_C-x_C*y_D)/(y_C-y_D)+1)/2)^2+(-(2*y_C*(((x_C^3-x_C^2+x_C-1)*y_C^2+x_C^5-3*x_C^4+3*x_C^3-x_C^2)*y_D^4+(((-3*x_C^2+2*x_C-1)*x_D-x_C^3+2*x_C^2-3*x_C+2)*y_C^3+((-3*x_C^4+10*x_C^3-11*x_C^2+4*x_C)*x_D-x_C^5+2*x_C^4-x_C^3)*y_C)*y_D^3+(((3*x_C-1)*x_D^2+(3*x_C^2-4*x_C+3)*x_D-x_C^2+2*x_C-1)*y_C^4+((3*x_C^3-13*x_C^2+14*x_C-4)*x_D^2+(3*x_C^4-4*x_C^3+x_C^2)*x_D-x_C^4+2*x_C^3-x_C^2)*y_C^2)*y_D^2+((-x_D^3+(2-3*x_C)*x_D^2+(2*x_C-2)*x_D)*y_C^5+((-x_C^2+8*x_C-6)*x_D^3+(-3*x_C^3+2*x_C^2-2*x_C+2)*x_D^2+(2*x_C^3-2*x_C^2)*x_D)*y_C^3)*y_D+(x_D^3-x_D^2)*y_C^6+(-2*x_D^4+(x_C^2+2)*x_D^3-x_C^2*x_D^2)*y_C^4))/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)+((y_C^2+(x_C-1)^2*x_C)*y_D^2-y_C*(y_C^2+2*(x_C-1)^2*x_D)*y_D+(x_C-1)*(x_D-1)*x_D*y_C^2)/(2*y_C*(y_D-y_C)*((x_C-1)*y_D-x_D*y_C)))^2-(1-((x_D*y_C-x_C*y_D)/(y_C-y_D)+1)/2)^2-(((y_C^2+(x_C-1)^2*x_C)*y_D^2-y_C*(y_C^2+2*(x_C-1)^2*x_D)*y_D+(x_C-1)*(x_D-1)*x_D*y_C^2)/(2*y_C*(y_D-y_C)*((x_C-1)*y_D-x_D*y_C)))^2=0,$

$$(((((x_C^2-1)*y_C^4+(2*x_C-2*x_C^2)*y_C^2-x_C^6+4*x_C^5-5*x_C^4+2*x_C^3)*y_D^4+((-2*x_C*x_D-2*x_C^2+2)*y_C^5+((2*x_C^3-2*x_C^2+2*x_C)*x_D-2*x_C^4+2*x_C^3+2*x_C^2-2*x_C)*y_C^3+((4*x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)+((y_C^2+(x_C-1)^2*x_C)*y_D^2-y_C*(y_C^2+2*(x_C-1)^2*x_D)*y_D+(x_C-1)*(x_D-1)*x_D*y_C^2)/(2*y_C*(y_D-y_C)*((x_C-1)*y_D-x_D*y_C)))^2=0,$$

$x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*$

$x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)-(x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2+(-(2*y_C*(((x_C^3-x_C^2+x_C-1)*y_C^2+x_C^5-3*x_C^4+3*x_C^3-x_C^2)*y_D^4+(((-3*x_C^2+2*x_C-1)*x_D-x_C^3+2*x_C^2-3*x_C+2)*y_C^3+((-3*x_C^4+10*x_C^3-11*x_C^2+4*x_C)*x_D-x_C^5+2*x_C^4-x_C^3)*y_C)*y_D^3+(((3*x_C-1)*x_D^2+(3*x_C^2-4*x_C+3)*x_D-x_C^2+2*x_C-1)*y_C^4+((3*x_C^3-13*x_C^2+14*x_C-4)*x_D^2+(3*x_C^4-4*x_C^3+x_C^2)*x_D-x_C^4+2*x_C^3-x_C^2)*y_C^2)*y_D^2+((-x_D^3+(2-3*x_C)*x_D^2+(2*x_C-2)*x_D)*y_C^5+((-x_C^2+8*x_C-6)*x_D^3+(-3*x_C^3+2*x_C^2-2*x_C+2)*x_D^2+(2*x_C^3-2*x_C^2)*x_D)*y_C^3)*y_D+(x_D^3-x_D^2)*y_C^6+(-2*x_D^4+(x_C^2+2)*x_D^3-x_C^2*x_D^2)*y_C^4))/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)-(y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2-((x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2-((y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2=0,$

and

$(((((x_C^2-1)*y_C^4+(2*x_C-2*x_C^2)*y_C^2-x_C^6+4*x_C^5-5*x_C^4+2*x_C^3)*y_D^4+((-2*x_C*x_D-2*x_C^2+2)*y_C^5+((2*x_C^3-2*x_C^2+2*x_C)*x_D-2*x_C^4+2*x_C^3+2*x_C^2-2*x_C)*y_C^3+((4*x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)-(y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2-((x_D*y_C-x_C*y_D)/(2*(y_C-y_D)))^2-((y_C/4-(x_C*((x_D*y_C-x_C*y_D)/(2*(y_C-y_D))-x_C/4))/y_C))^2=0,$

and

$((((x_C^2-1)*y_C^4+(2*x_C-2*x_C^2)*y_C^2-x_C^6+4*x_C^5-5*x_C^4+2*x_C^3)*y_D^4+((-2*x_C*x_D-2*x_C^2+2)*y_C^5+((2*x_C^3-2*x_C^2+2*x_C)*x_D-2*x_C^4+2*x_C^3+2*x_C^2-2*x_C)*y_C^3+((4*x_C^5-18*x_C^4+22*x_C^3-8*x_C^2)*x_D+2*x_C^5-4*x_C^4+2*x_C^3)*y_C)*y_D^3+((x_D^2+4*x_C*x_D+x_C^2-1)*y_C^6+((-4*x_C^2+4*x_C-2)*x_D^2+(2*x_C^3-2*x_C^2+2*x_C-2)*x_D+2*x_C^4-2*x_C^3)*y_C^4+((-5*x_C^4+28*x_C^3-30*x_C^2+8*x_C)*x_D^2+(-2*x_C^5-2*x_C^4+6*x_C^3-2*x_C^2)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+((-2*x_D^2-2*x_C*x_D)*y_C^7+((2*x_C-2)*x_D^3+(2*x_C^2-2*x_C)*x_D^2+(-4*x_C^3+4*x_C^2-4*x_C+2)*x_D)*y_C^5+((2*x_C^3-18*x_C^2+12*x_C)*x_D^3+(4*x_C^4-2*x_C^3+4*x_C^2-4*x_C)*x_D^2+(-2*x_C^5+4*x_C^4-4*x_C^3+2*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2-2*x_C)*x_D^3+(2*x_C^2-2*x_C+2)*x_D^2)*y_C^6+(4*x_C*x_D^4+(-2*x_C^3+2*x_C^2-4*x_C)*x_D^3+(x_C^4-2*x_C^3+2*x_C^2)*x_D^2)*y_C^4)/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*$

$x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)-(((x_C+1)*y_C^2+x_C^3+x_C^2-2*x_C)*y_D-x_D*y_C^3+(-x_C^2-2)*x_D*y_C)/(4*((x_C-1)*y_D-x_D*y_C)))^2+(-(2*y_C*(((x_C^3-x_C^2+x_C-1)*y_C^2+x_C^5-3*x_C^4+3*x_C^3-x_C^2)*y_D^4+(((-3*x_C^2+2*x_C-1)*x_D-x_C^3+2*x_C^2-3*x_C+2)*y_C^3+((-3*x_C^4+10*x_C^3-11*x_C^2+4*x_C)*x_D-x_C^5+2*x_C^4-x_C^3)*y_C)*y_D^3+(((3*x_C-1)*x_D^2+(3*x_C^2-4*x_C+3)*x_D-x_C^2+2*x_C-1)*y_C^4+((3*x_C^3-13*x_C^2+14*x_C-4)*x_D^2+(3*x_C^4-4*x_C^3+x_C^2)*x_D-x_C^4+2*x_C^3-x_C^2)*y_C^2)*y_D^2+((-x_D^3+(2-3*x_C)*x_D^2+(2*x_C-2)*x_D)*y_C^5+((-x_C^2+8*x_C-6)*x_D^3+(-3*x_C^3+2*x_C^2-2*x_C+2)*x_D^2+(2*x_C^3-2*x_C^2)*x_D)*y_C^3)*y_D+(x_D^3-x_D^2)*y_C^6+(-2*x_D^4+(x_C^2+2)*x_D^3-x_C^2*x_D^2)*y_C^4))/(((x_C^2+2*x_C+1)*y_C^4+(2*x_C^4-4*x_C^3+2*x_C^2-4*x_C+4)*y_C^2+x_C^6-6*x_C^5+13*x_C^4-12*x_C^3+4*x_C^2)*y_D^4+(((-2*x_C-2)*x_D-2*x_C^2-4*x_C-2)*y_C^5+((-4*x_C^3+16*x_C^2-8*x_C)*x_D-4*x_C^4+4*x_C^3-4*x_C^2+12*x_C-8)*y_C^3+((-2*x_C^5+18*x_C^4-48*x_C^3+48*x_C^2-16*x_C)*x_D-2*x_C^6+8*x_C^5-10*x_C^4+4*x_C^3)*y_C)*y_D^3+((x_D^2+(4*x_C+4)*x_D+x_C^2+2*x_C+1)*y_C^6+((2*x_C^2-16*x_C+8)*x_D^2+(8*x_C^3-20*x_C^2+16*x_C-4)*x_D+2*x_C^4+2*x_C^2-8*x_C+4)*y_C^4+((x_C^4-16*x_C^3+56*x_C^2-56*x_C+16)*x_D^2+(4*x_C^5-24*x_C^4+40*x_C^3-28*x_C^2+8*x_C)*x_D+x_C^6-2*x_C^5+x_C^4)*y_C^2)*y_D^2+(((-2*x_C-2)*x_D-2*x_D^2)*y_C^7+(4*x_D^3+(-4*x_C^2+20*x_C-16)*x_D^2+(-4*x_C^3+4*x_C^2-8*x_C+4)*x_D)*y_C^5+((4*x_C^2-24*x_C+16)*x_D^3+(-2*x_C^4+20*x_C^3-40*x_C^2+40*x_C-16)*x_D^2+(-2*x_C^5+6*x_C^4-8*x_C^3+4*x_C^2)*x_D)*y_C^3)*y_D+x_D^2*y_C^8+((2*x_C^2-4*x_C+8)*x_D^2-4*x_D^3)*y_C^6+(4*x_D^4+(-4*x_C^2+8*x_C-8)*x_D^3+(x_C^4-4*x_C^3+8*x_C^2-8*x_C+4)*x_D^2)*y_C^4)+(((x_C^2-2*x_C+3)*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_D-(x_C-3)*x_D*y_C^3-(x_C-2)*(x_C-1)*x_C*x_D*y_C)/(4*y_C*((x_C-1)*y_D-x_D*y_C)))^2-(x_C-(((x_C+1)*y_C^2+x_C^3+x_C^2-2*x_C)*y_D-x_D*y_C^3+(-x_C^2-2)*x_D*y_C)/(4*((x_C-1)*y_D-x_D*y_C)))^2-(y_C+(((x_C^2-2*x_C+3)*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_D-(x_C-3)*x_D*y_C^3-(x_C-2)*(x_C-1)*x_C*x_D*y_C)/(4*y_C*((x_C-1)*y_D-x_D*y_C)))^2 = 0$

as desired. □

References

[1] Ehrmann, J. P., Forum Geometricorum Volume 4 (2004) 35–52.

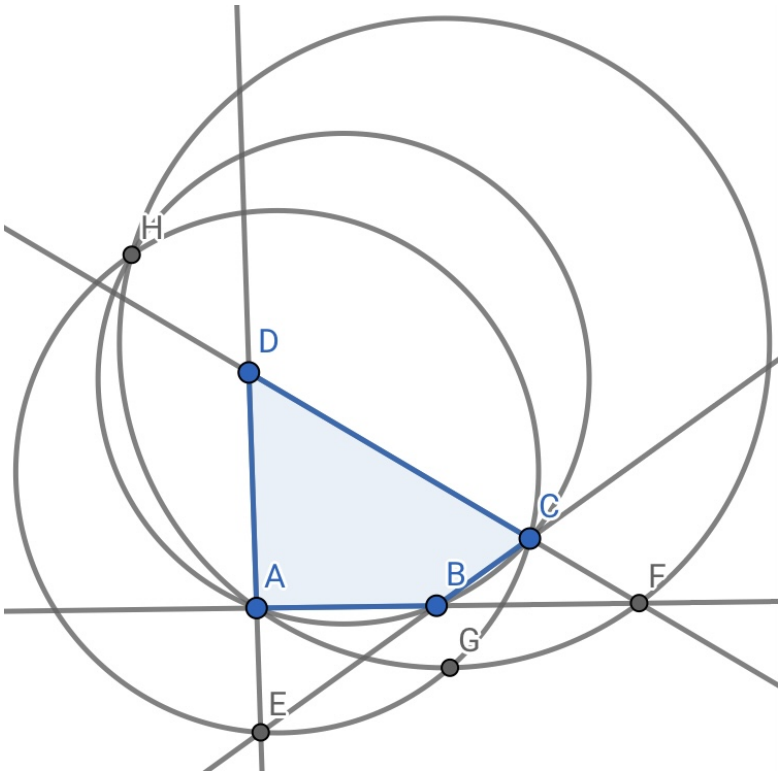

Figure 1. Illustration of theorem 2.1.

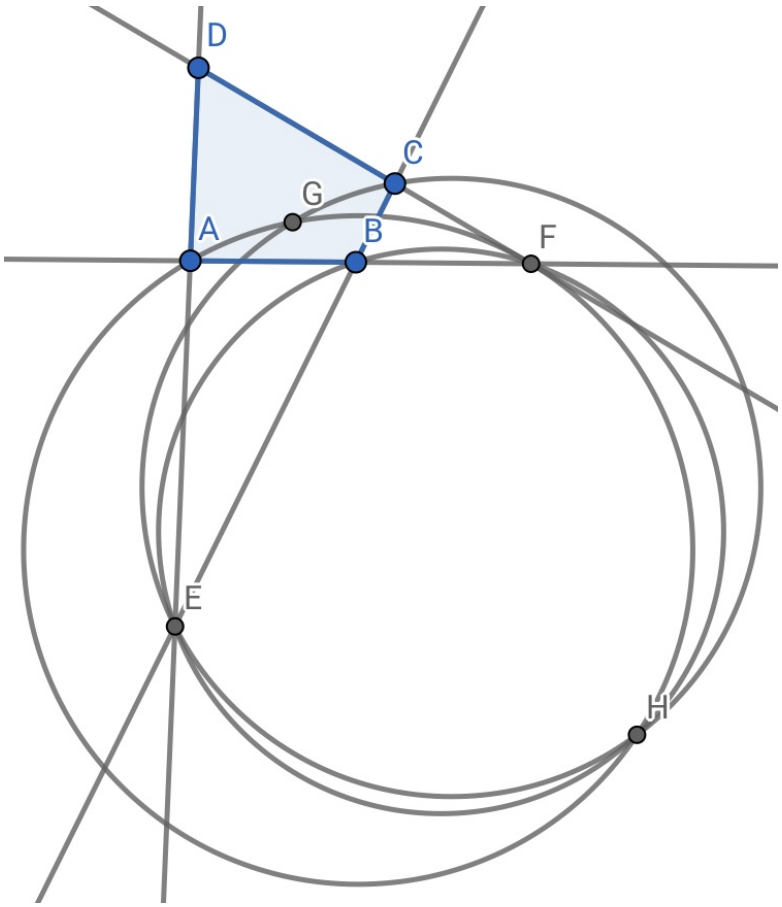

Figure 2. Illustration of theorem 2.2.