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The Orthocenter is the Power Center of Three Certain Circles

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1. Introduction

Pick three points on the different sides of a triangle. We prove that the circles whose diameters are the cevians related to these points have the orthocenter as their power center.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be a point on line AB.
Let E be a point on line AC.
Let F be a point on line BC.
Let α be the circle whose diameter is AF.
Let β be the circle whose diameter is BE.
Let γ be the circle whose diameter is CD.
Then the power center of α, β, and γ is the orthocenter of A, B, and C.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates $(x_D, 0)$.
E has coordinates $(x_E, \frac{x_E y_C}{x_C})$.

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F has coordinates $(x_F, (x_F - 1)(\frac{y_C}{x_C - 1}))$.
The orthocenter of A, B, and C has coordinates $(x_C, \frac{x_C(1-x_C)}{y_C})$.
α is given by the equation
$(x - x_F/2)^2 + (y - (x_F - 1)(\frac{y_C}{2(x_C-1)}))^2 - (x_F/2)^2 - ((x_F - 1)(\frac{y_C}{2(x_C-1)}))^2 = 0.$
β is given by the equation
$(x - (x_E + 1)/2)^2 + (y - \frac{x_E y_C}{2x_C})^2 - (1 - (x_E + 1)/2)^2 - (\frac{x_E y_C}{2x_C})^2 = 0.$
γ is given by the equation
$(x - (x_C + x_D)/2)^2 + (y - y_C/2)^2 - (x_D - (x_C + x_D)/2)^2 - (y_C/2)^2 = 0.$
The radical line between α and β is given by the equation
$-(x_F - x_E - 1) * x - ((x_C * x_F - (x_C - 1) * x_E - x_C) * y * y_C)/((x_C - 1) * x_C) - x_E = 0.$
The radical line between α and γ is given by the equation
$-(x_F - x_D - x_C) * x - ((x_F - x_C) * y * y_C)/(x_C - 1) - x_C * x_D = 0.$
The radical line between β and γ is given by the equation
$-(x_E - x_D - x_C + 1) * x - ((x_E - x_C) * y * y_C)/x_C + x_E - x_C * x_D = 0.$
We then verify that the orthocenter lies on these lines since we calculate that
$-(x_F - x_E - 1) * x_C - ((x_C * x_F - (x_C - 1) * x_E - x_C) * (\frac{x_C(1-x_C)}{y_C}) * y_C)/((x_C - 1) * x_C) - x_E = 0,$
$-(x_F - x_D - x_C) * x_C - ((x_F - x_C) * (\frac{x_C(1-x_C)}{y_C}) * y_C)/(x_C - 1) - x_C * x_D = 0,$
and
$-(x_E - x_D - x_C + 1) * x_C - ((x_E - x_C) * (\frac{x_C(1-x_C)}{y_C}) * y_C)/x_C + x_E - x_C * x_D = 0$
as desired. □

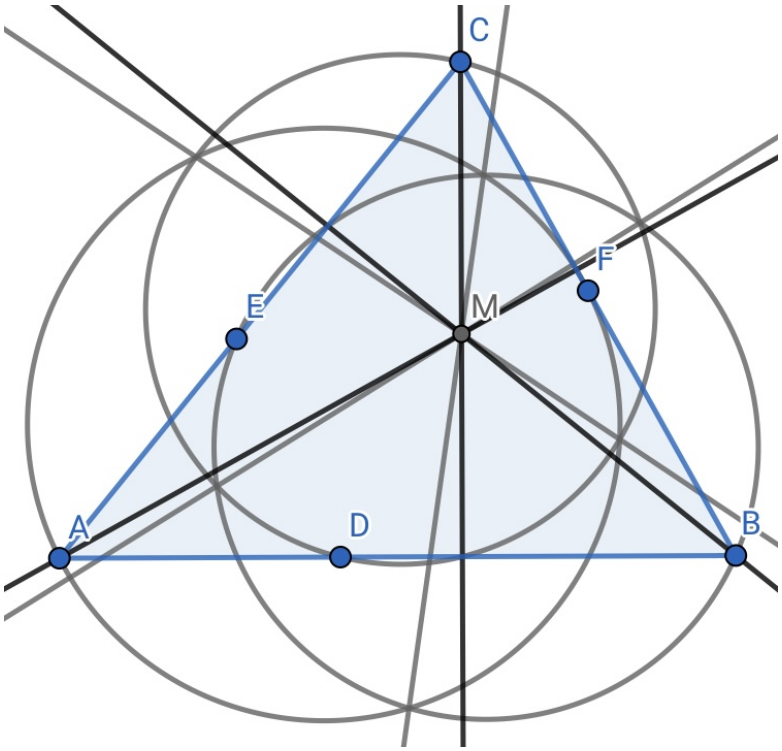

FIGURE 1. Illustration.

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