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Some Properties Regarding a Certain Circle Associated to a Cyclic Quadrilateral

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Abstract. More results are introduced concerning certain circles associated to cyclic quadrilaterals.

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1. Introduction

In [1], it was shown that for a given cyclic quadrilateral, if lines are extended on opposite sides to intersect at a point, then this point, the center of the circumcircle, and the midpoints of the respective sides are concyclic. Here, we carry on and find a handful of more results related to contructions involving those circles.

2. Preliminaries

Let $\square ABCD$ be a cyclic quadrilateral (arranged counterclockwise).
Let E be the midpoint of A and B, and F be the midpoint of C and D.
Let ζ denote the circumcircle of $\square ABCD$.
Let G denote the center of ζ.
Let the extended lines AB and CD intersect at H.
Let η denote the circle passing through E, F, G, and H.
Let ζ and η intersect at I and J.
Let the diagonals of $\square ABCD$ intersect at K.
Let L be the midpoint of A and D.
Let M be the midpoint of B and C.
Let the extended lines AD and BC intersect at N.

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Let θ denote the circle passing through G, L, M, and N.
Let ζ and θ intersect at O and P.
Let lines IJ and GH intersect at Q.
Let lines GN and HK intersect at R.
Let η and θ intersect at G and S.
Let T denotethe center of the circle passing through A, D, G, and R.
Let U denote the center of the circle passingthrough A, B, R, and K.
Let V denote the center of the circle passingthrough B,C, K, and Q.
Let W denote the center of the circle passing through C, D, G, and Q. Let Z denote the center of the circle passing through G, K, Q, and R. Let the line AC intersect the circle passing through A, D, G, and R at A_1. Let the circle centered at V intersect θ at Q and B_1.
Let line FG intersect line IJ at point C_1.

3. Results

Theorem 3.1. *The line segment between G and H is the diameter of δ.*

It immediately follows from this that the lines GH and IJ intersect at a right angle.

Theorem 3.2. *I, J, and K are collinear.*

Theorem 3.3. *The extended lines AD, BC, and IJ intersect at a point N. This point is on the opposite side of G on the circle passing through G, the midpoint of A and D, and the midpoint of B and C.*

Theorem 3.4. *H, K, O, and P are collinear.*

Theorem 3.5. *Q lies on θ.*

Theorem 3.6. *R lies on η.*

Theorem 3.7. *G, K, Q, and R are concyclic.*

Theorem 3.8. *H, N, and S are collinear.*

Theorem 3.9. *A, D, G, and R are concyclic.*

Theorem 3.10. *A, B, R, and K are concyclic.*

Theorem 3.11. *B, C, K, and Q are concyclic.*

Theorem 3.12. *C, D, G, and Q are concyclic.*

Theorem 3.13. *G, K, Q, and R are concyclic.*

Theorem 3.14. *A, D, G, and R at A_1 are collinear.*

Theorem 3.15. *G, K, S, and Z are collinear.*

Theorem 3.16. *K, L, and B_1 are collinear.*

Theorem 3.17. *G, E, and U are collinear.*

Theorem 3.18. *F, G, W, A_1, and C_1 are collinear.*

Theorem 3.19. *U, W, and N are collinear.*

Theorem 3.20. *U, the center of η and the center of θ are collinear.*

Theorem 3.21. *T, V, and H are collinear.*

Theorem 3.22. *G, L, and T are collinear.*

Theorem 3.23. *G, M, and V are collinear.*

Theorem 3.24. *A, Q, and R are collinear.*

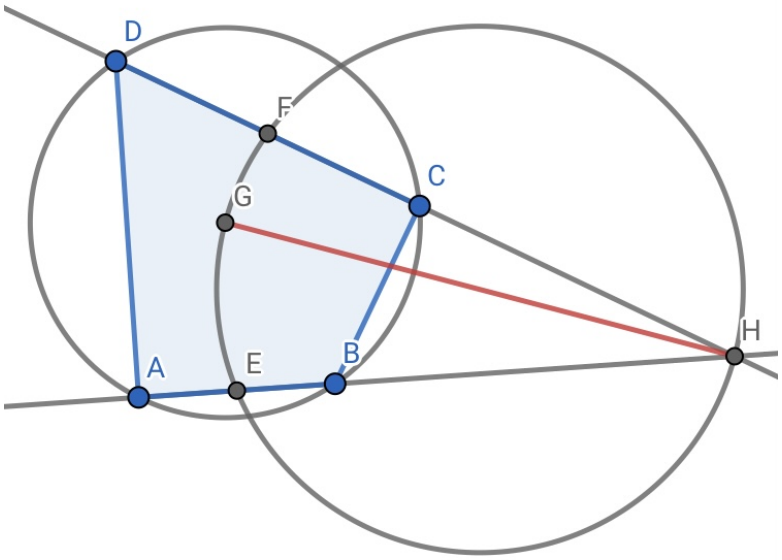

Figure 1. The line between G and H (in red) is the diameter of the circle passing through E, F, G, and H.

References

[1] Warren, B.L, Some Sets of Concyclic Points and a Point Where Several Particular Circles Intersect. (To Appear).

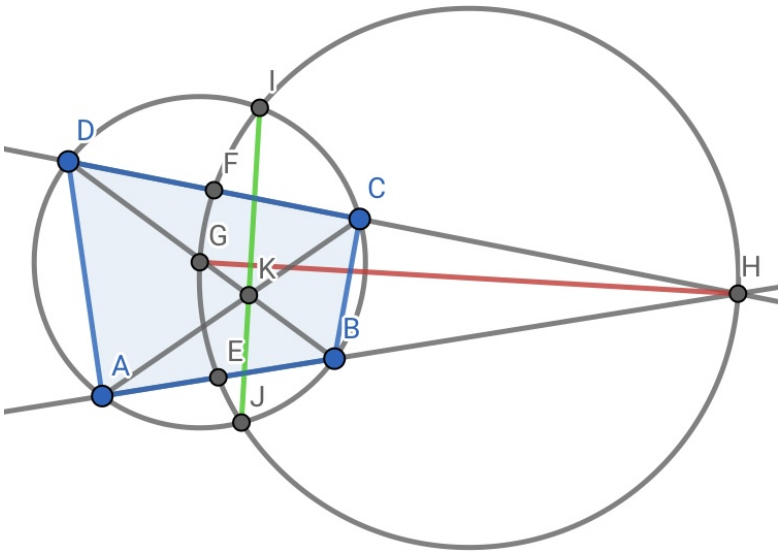

Figure 2. The point of intersection of the diagonals of $\square ABCD$ and the points of intersection of the circumcircle of $\square ABCD$ and the circle passing through E, F, G, and H are collinear.

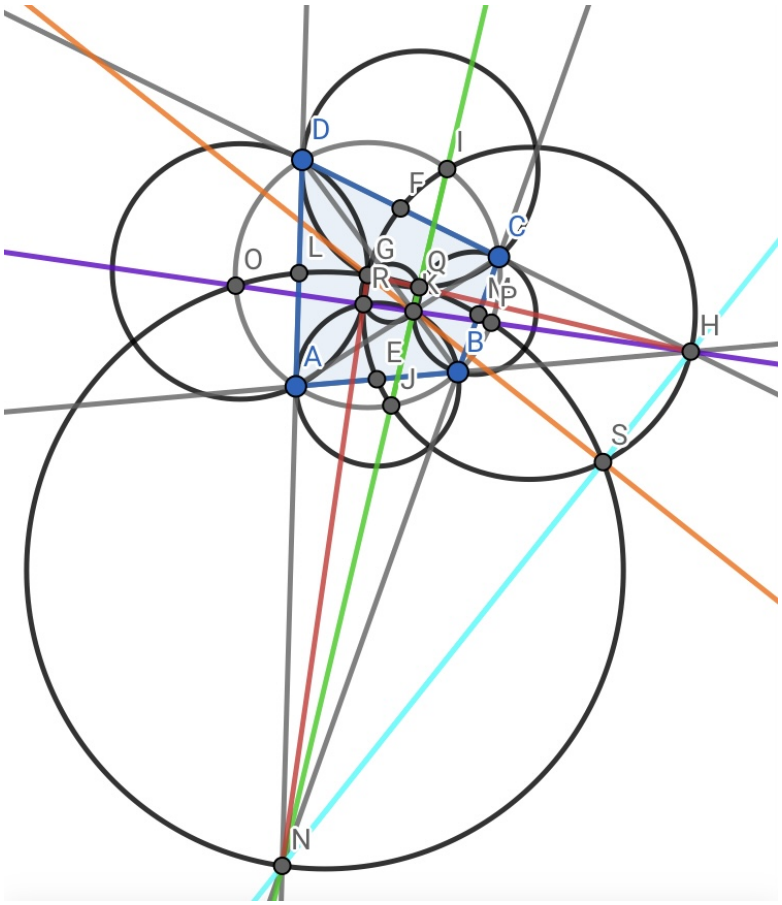

Figure 3.

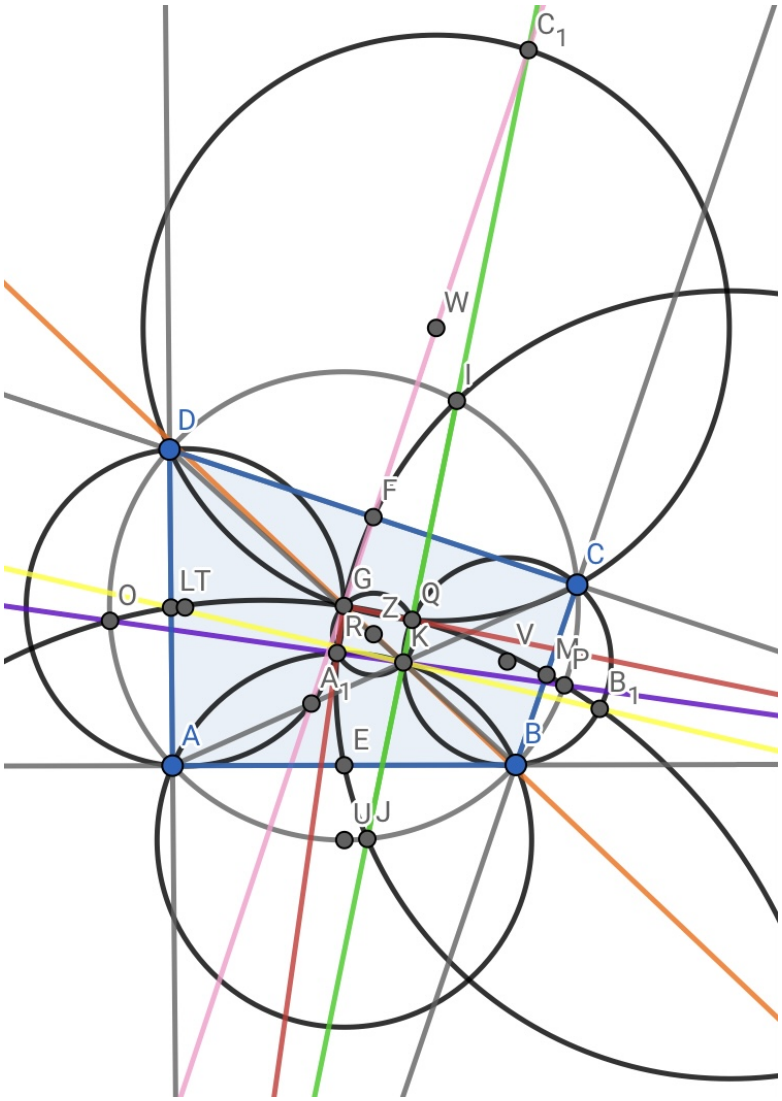

Figure 4.