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Another Point that Lies on the Newton Line

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Abstract. We show in a certain construction of a given quadrilateral that the concurrence of several Newton lines of smaller quadrilaterals concur at a point that lies on the Newton line of the main quadrilateral.

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1. Introduction

Newton's theorem [1] states that in a given non-rhomic tangential quadrilateral, the center of the incircle lies on the Newton line (the line formed by the midoints of the diagonals).
Here we construct a quadrilateral, pick and point in the plane, and form four quadrilaterals between that point, the vertices, and the midpoints of the quadrilateral. If we then draw the Newton lines of these smaller quadrilaterals and form the diagonals of the parallelogram formed by the intersections of the Newton lines, it turns out that the point where these diagonals intersect lies on the Newton line of the original quadrilateral.

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2. Results

Theorem 2.1. *Let $\square ABCD$ be any given quadrilateral.*
Let E be the midpoint of A and B, F be the midpoint of B and C, G be the midpoint of C and D, and H be the midpoint of A and D.
Let I be any given point.
Let the Newton Lines of $\square AEIH$, $\square BFIE$, $\square CGIF$, and $\square GDHI$ intersect at points R, S, T, and U.
Let the diagonals of $\square RSTU$ intersect at point V.
Then V lies on the Newton line of $\square ABCD$.

Proof. Let J be the midpoint of A and I, and let K be the midpoint of E and H.
Let L be the midpoint of E and F, and let M be the midpoint of B and I.
Let N be the midpoint of C and I, and let O be the midpoint of F and G.
Let P be the midpoint of G and H, and let Q be the midpoint of D and I.
Let W be the midpoint of A and C, and let Z be the midpoint of B and D.
Let A have cartesian coordinates given by (x_A, y_A).
Let B have cartesian coordinates given by (x_B, y_B).
Let C have cartesian coordinates given by (x_C, y_C).
Let D have cartesian coordinates given by (x_D, y_D).
Let I have cartesian coordinates given by (x_I, y_I).
E has coordinates given by $(\frac{1}{2}(x_A + x_B), \frac{1}{2}(y_A + y_B))$.
F has coordinates given by $(\frac{1}{2}(x_B + x_C), \frac{1}{2}(y_B + y_C))$.
G has coordinates given by $(\frac{1}{2}(x_C + x_D), \frac{1}{2}(y_C + y_D))$.
H has coordinates given by $(\frac{1}{2}(x_A + x_D), \frac{1}{2}(y_A + y_D))$.
J has coordinates given by $(\frac{1}{2}(x_A + x_I), \frac{1}{2}(y_A + y_I))$.
K has coordinates given by $(\frac{1}{2}(x_E + \frac{1}{2}(x_A + x_D)), \frac{1}{2}(y_E + \frac{1}{2}(y_A + y_D)))$.
L has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_A+x_B)+\frac{1}{2}(x_B+x_C)), \frac{1}{2}(\frac{1}{2}(y_A+y_B)+\frac{1}{2}(y_B+y_C)))$.
M has coordinates given by $(\frac{1}{2}(x_B + x_I), \frac{1}{2}(y_B + y_I))$.
N has coordinates given by $(\frac{1}{2}(x_C + x_I), \frac{1}{2}(y_C + y_I))$.
O has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_B+x_C)+\frac{1}{2}(x_C+x_D)), \frac{1}{2}(\frac{1}{2}(y_B+y_C)+\frac{1}{2}(y_C+y_D)))$.
P has coordinates given by $(\frac{1}{2}(\frac{1}{2}(x_C+x_D)+\frac{1}{2}(x_A+x_D)), \frac{1}{2}(\frac{1}{2}(y_C+y_D)+\frac{1}{2}(y_A+y_D)))$.
Q has coordinates given by $(\frac{1}{2}(x_D + x_I), \frac{1}{2}(y_D + y_I))$.
R has coordinates given by
$(\frac{y_J-y_L+x_L(\frac{y_L-y_M}{x_L-x_M})-x_J(\frac{y_J-y_K}{x_J-x_K})}{\frac{y_L-y_M}{x_L-x_M}-\frac{y_J-y_K}{x_J-x_K}}, y_J + (\frac{y_J-y_L+x_L(\frac{y_L-y_M}{x_L-x_M})-x_J(\frac{y_J-y_K}{x_J-x_K})}{\frac{y_L-y_M}{x_L-x_M}-\frac{y_J-y_K}{x_J-x_K}} - x_J)(\frac{y_J-y_K}{x_J-x_K}))$.
S has coordinates given by
$(\frac{y_N-y_L+x_L(\frac{y_L-y_M}{x_L-x_M})-x_N(\frac{y_N-y_O}{x_N-x_O})}{\frac{y_L-y_M}{x_L-x_M}-\frac{y_N-y_O}{x_N-x_O}}, y_N + (\frac{y_N-y_L+x_L(\frac{y_L-y_M}{x_L-x_M})-x_N(\frac{y_N-y_O}{x_N-x_O})}{\frac{y_L-y_M}{x_L-x_M}-\frac{y_N-y_O}{x_N-x_O}} - x_N)(\frac{y_N-y_O}{x_N-x_O}))$.
T has coordinates given by
$(\frac{y_N-y_P+x_P(\frac{y_P-y_Q}{x_P-x_Q})-x_N(\frac{y_N-y_O}{x_N-x_O})}{\frac{y_P-y_Q}{x_P-x_Q}-\frac{y_N-y_O}{x_N-x_O}}, y_N + (\frac{y_N-y_P+x_P(\frac{y_P-y_Q}{x_P-x_Q})-x_N(\frac{y_N-y_O}{x_N-x_O})}{\frac{y_P-y_Q}{x_P-x_Q}-\frac{y_N-y_O}{x_N-x_O}} - x_N)(\frac{y_N-y_O}{x_N-x_O}))$.
U has coordinates given by
$(\frac{y_J-y_P+x_P(\frac{y_P-y_Q}{x_P-x_Q})-x_J(\frac{y_J-y_K}{x_J-x_K})}{\frac{y_P-y_Q}{x_P-x_Q}-\frac{y_J-y_K}{x_J-x_K}}, y_J + (\frac{y_J-y_P+x_P(\frac{y_P-y_Q}{x_P-x_Q})-x_J(\frac{y_J-y_K}{x_J-x_K})}{\frac{y_P-y_Q}{x_P-x_Q}-\frac{y_J-y_K}{x_J-x_K}} - x_J)(\frac{y_J-y_K}{x_J-x_K}))$.
One can then verify that V has coordinates given by
$(\frac{1}{2}(\frac{1}{2}(x_A + x_C) + \frac{1}{2}(x_B + x_D)), \frac{1}{2}(\frac{1}{2}(y_A + y_C) + \frac{1}{2}(y_B + y_D)))$
which is the midpoint of W and Z, so indeed V lies on the Newton line of $\square ABCD$ (moreover V is the midpoint of the diagonals of $\square ABCD$). $\square$

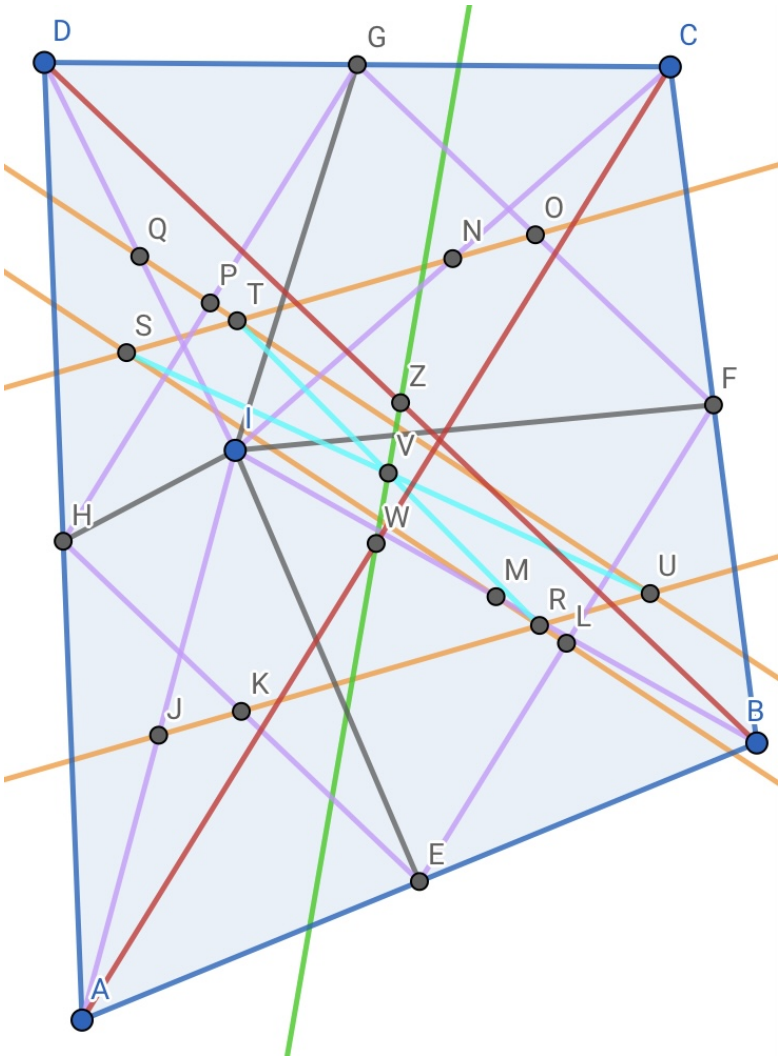

Figure 1. The Newton lines (in orange) of $\square AEIH$, $\square BFIE$, $\square CGIF$, and $\square GDHI$ intersect at points R, S, T, and U, and the point V where the diagonals of $\square RSTU$ meet lies on the Newton line (in green) of $\square ABCD$.

References

[1] Alsina, C., Nelson, R.B., Charming Proofs: A Journey into Elegant Mathematics. *Mathematics Association of America*, (2010), pp.117-118.