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Congruent Circles On Locus Problems

Abdilkadir Altıntaş [2]
Emirdağ,
Afyon-TURKEY
e-mail: kadiraltintas1977@gmail.com

Abstract. By using the computer program "Mathematica" and "Geogebra", we give theorems about congruent circles related to triangle cubic and higher degree curves. Mathematica computations are made by "baricentricas.m" file written by Francisco Javier García Capitán.

Keywords. Triangle geometry, Computer-discovered mathematics, Euclidean geometry, Reflection, Napoleon-Feuerbach Cubic, Lucas Cubic, Neuberg Cubic, Congruent Circles, Barycentric Coordinates.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

K001 is the isogonal pK with pivot $X(30)$ = infinite point of the Euler line : The Euler lines of triangles PBC, PCA, PAB concur (on the Euler line) if and only if P lies on the Neuberg cubic (together with $C(O,R)$ and line at infinity) [2]. Barycentric equation of the curve:

$$\sum_{cyclic} \left[a^2\left(b^2+c^2\right)+(b^2-c^2)^2-2a^4\right)\right] x(c^2y^2-b^2z^2)=0$$

K005, the Napoleon-Feuerbach cubic is the isogonal pK with pivot $X(5)$ = nine-point center [2]. Barycentric equation of the curve:

$$\sum_{cyclic} \left[a^2\left(b^2+c^2\right)-(b^2-c^2)^2\right)\right] x(c^2y^2-b^2z^2)=0$$

[2]Corresponding author

K007, the Lucas cubic is the isotomic pK with pivot $X(69)$, isotomic conjugate of orthocenter H [2]. Barycentric equation of the curve:

$$\sum_{cyclic} \left(b^2+c^2-a^2\right) x(y^2-z^2)=0$$

Some locus properties of K001, K005, K007 listed at [1],[2],[4].

The Euler line is the line HO passing through the orthocenter H and circumcenter O of triangle ABC [5].

The Brocard axis is the line KO passing through the symmedian point K and circumcenter O of triangle ABC [6].

2. THEOREMS

Theorem 2.1. *Let P be a point. DEF cevian triangle of P. A_b, A_c are reflections of A on BE, CF respectively. Define B_a, B_c, C_a, C_b cyclically. Circles $\Gamma_A : (AB_cC_b), \Gamma_B : (BC_aA_c), \Gamma_C : (CB_aA_b)$ are congruent iff P lies on K005-Napoleon Feuerbach Cubic of ABC (Figure 1).*

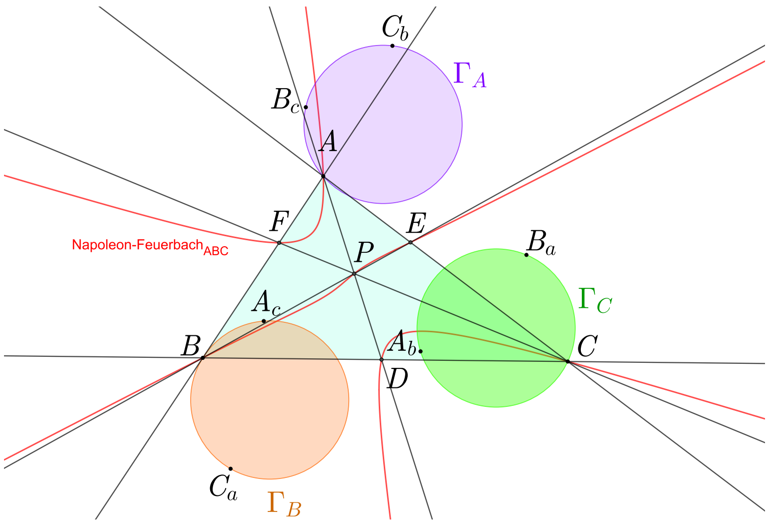

FIGURE 1.

Proof. Mathematica computations gives the following locus:

$$\sum_{cyclic} \left[a^2\left(b^2+c^2\right)-(b^2-c^2)^2\right)\right] x(c^2y^2-b^2z^2)=0$$

which is K005-Napoleon Feuerbach Cubic ABC. □

Theorem 2.2. *Let P be a point. DEF cevian triangle of P. C_D, B_D are reflections of C, B on D respectively. Define A_E, C_E, A_F, B_F cyclically. Circles $(AB_FC_E), (BC_DA_F), (CA_EB_D)$ are congruent iff P lies on K007-Lucas Cubic of ABC (Figure 2).*

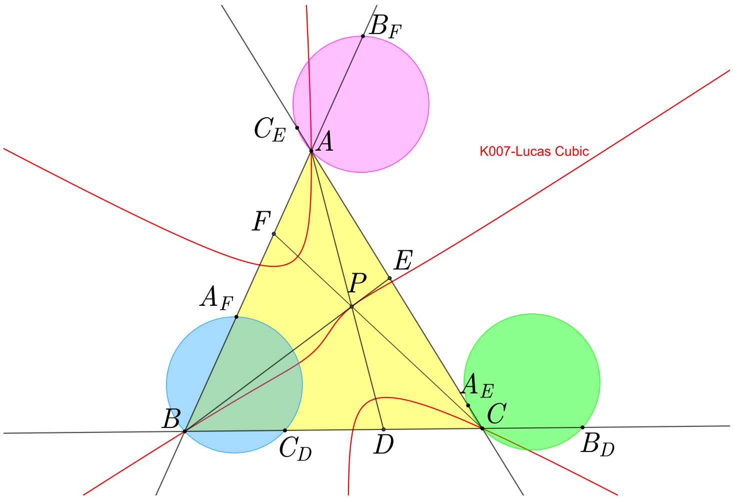

FIGURE 2.

Proof. Mathematica computations gives the following locus:

$$\sum_{cyclic} \left(b^2 + c^2 - a^2\right) x(y^2 - z^2) = 0$$

which is K007-Lucas Cubic of ABC. □

Theorem 2.3. *Let P be a point. B_A, C_A are reflections of C, B on Euler line of PBC. Define A_B, C_B, A_C, A_B cyclically. Circles $(AB_CC_B), (BC_AA_C), (CA_BB_A)$ are congruent iff P lies on Neuberg Cubic of ABC (Figure 3).*

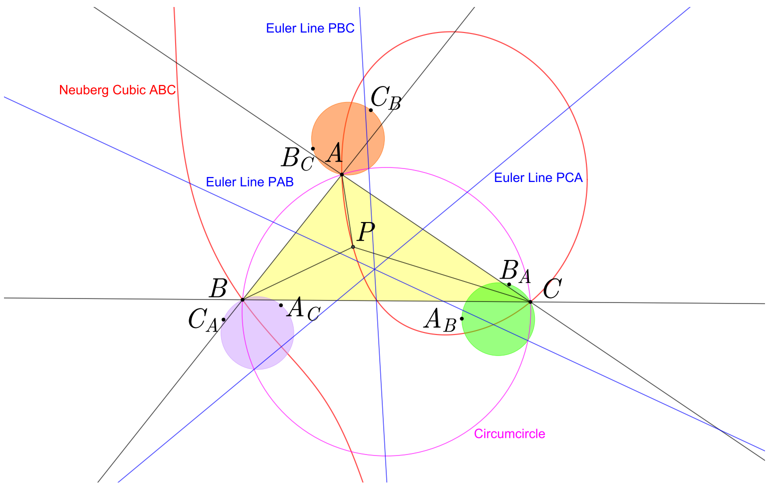

FIGURE 3.

Proof. Mathematica computations gives the following locus:

$$\sum_{cyclic} \left[a^2\left(b^2 + c^2\right) + (b^2 - c^2)^2 - 2a^4\right] x(c^2y^2 - b^2z^2) = 0$$

which is K001-Neuberg Cubic of ABC. □

Theorem 2.4. *Let P be a point. B_A, C_A are reflections of C, B on Brocard axis of PBC. Define A_B, C_B, A_C, A_B cyclically. Circles $(AB_CC_B), (BC_AA_C), (CA_BB_A)$ are congruent iff P lies on Neuberg Cubic of ABC.*

Proof. Mathematica computations gives the following locus:

$$\sum_{cyclic} \left[a^2\left(b^2+c^2\right)+(b^2-c^2)^2-2a^4\right)\right] x(c^2y^2-b^2z^2)=0$$

which is K001-Neuberg Cubic of ABC. □

Conjecture. Let ABC be triangle, P be a point. $\mathcal{L_A}, \mathcal{L_B}, \mathcal{L_C}$ be same lines passing through circumcenters of PBC, PCA, PAB. B_A, C_A are reflections of B, C on $\mathcal{L_A}$. Define A_B, C_B, A_C, A_B cyclically. Iff $\mathcal{L_A}, \mathcal{L_B}, \mathcal{L_C}$ are concurrent then circles $(AB_CC_B), (BC_AA_C), (CA_BB_A)$ are congruent.

Some congruent circles belongs to higher degree locus problems are listed below:

1. Let ABC be a triangle, $G = X_2$-centroid of ABC. DEF cevian triangle of G. O_A, O_B, O_C are circumcenters of AEF, BFD, CDE respectively. O_AB, O_AC reflections of O_A on cevians BE, CF. Define O_BA, O_BC, O_CA, O_CB cyclically. Circles $(O_AO_BCO_CB), (O_BO_ACO_CA), (O_CO_ABO_BA)$ are congruent [7].

2. Let ABC be a triangle. DEF cevian triangle of X_7-Gergonne point of ABC. HO_A, H_B, H_C are orthocenters of AEF, BFD, CDE respectively. H_AB, H_AC reflections of H_A on cevians BE, CF. Define H_BA, H_BC, H_CA, H_CB cyclically. Circles $(H_AH_BCH_CB), (H_BH_ACHO_CA), (H_CH_ABH_BA)$ are congruent [9].

3. Let ABC be a triangle. DEF cevian triangle of X_7-Gergonne point of ABC. X_A, X_B, X_C are 1st Fermat points of AEF, BFD, CDE respectively. X_AB, X_AC reflections of X_A on cevians BE, CF. Define X_BA, X_BC, X_CA, X_CB cyclically. Circles $(X_AX_BCX_CB), (X_BX_ACXO_CA), (X_CX_ABX_BA)$ are congruent [11].

4. Let ABC be a triangle. DEF orthic triangle of ABC. X_A, X_B, X_C are X_{74}-Isogonoal conjugate of Euler infinity points of AEF, BFD, CDE respectively. X_AB, X_AC reflections of X_A on cevians BE, CF. Define X_BA, X_BC, X_CA, X_CB cyclically. Circles $(X_AX_BCX_CB), (X_BX_ACXO_CA), (X_CX_ABX_BA)$ are congruent.

5. Let ABC be a triangle. DEF-cevian triangle of X_1-incenter of ABC. G_A, G_B, G_C are centroids of AEF, BFD, CDE respectively. G_AB, G_AC reflections of G_A on cevians BE, CF. Define G_BA, G_BC, G_CA, G_CB cyclically. Circles $(G_AG_BCG_CB), (G_BG_ACGO_CA), (G_CG_ABG_BA)$ are congruent [10].

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