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On Concurrent Euler Lines

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Abstract. By using the computer program "Mathematica" and "Geogebra" we give theorems about concurrent Euler Lines.

Keywords. Triangle geometry, Computer-Discovered Mathematics, Euclidean geometry, Euler Line, Barycentric Coordinates.

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1. Introduction

In geometry, the Euler line, named after Leonhard Euler, is a line determined from any triangle that is not equilateral. It is a central line of the triangle [3]. The line on which the orthocenter H, triangle centroid G, circumcenter O, de Longchamps point L, nine-point center N, and a number of other important triangle centers lie is Euler line of ABC [2]. The Euler line is perpendicular to the de Longchamps line and orthic axis. Some Kimberling centers X_i lying on the line include $i = 2$ (triangle centroid G), 3 (circumcenter O), 4 (orthocenter H), 5 (nine-point center N), 20 (de Longchamps point L), 21 (Schiffler point), 22 (Exeter point), 23 (far-out point), $24, 25, 26, 27, 28, 29, 30$, (Euler infinity point).

2. Theorems

Theorem 2.1. *Let $H = X(4)$- orthocenter of ABC. DEF circumcevian triangle of H. A_1, A_2 orthogonal projections of D on AB, AC respectively. Define B_1, B_2, C_1, C_2 cyclically.Euler lines of DA_1A_2, EB_1B_2, FC_1C_2 concur. Concurrency point is $X(32352)$=Orthologic center of Anti-Wasat to Hatzipolakis-Moses (Figure 1).*

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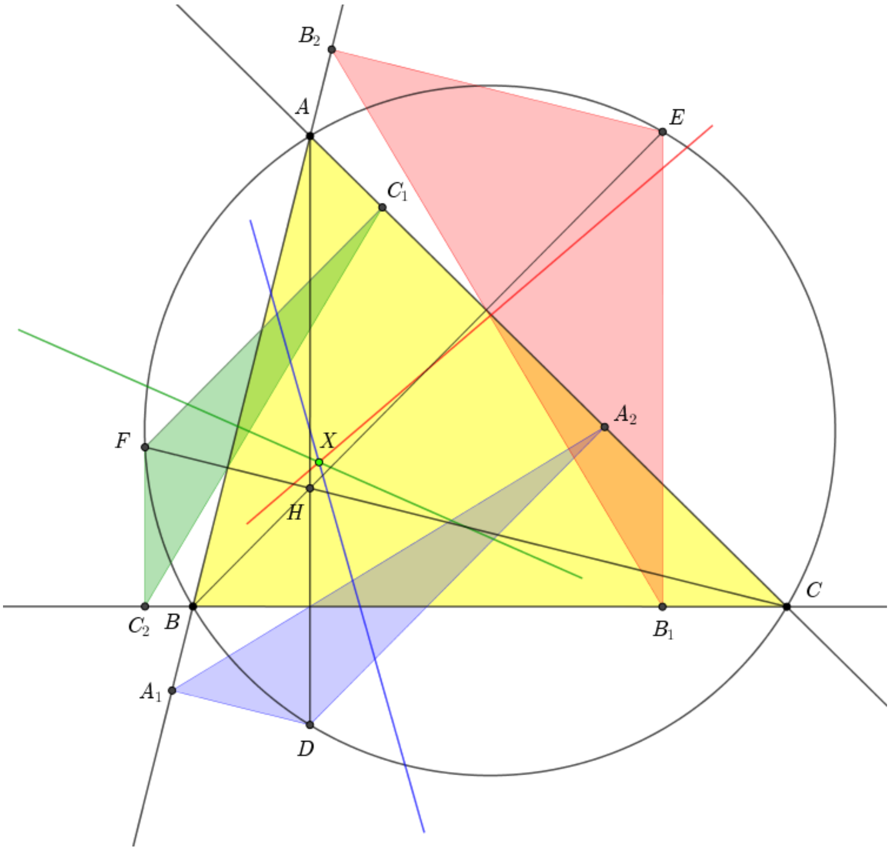

FIGURE 1.

Theorem 2.2. *Let $I = X(1)$ be incenter of ABC and DEF circumcevian triangle of I. A_1, A_2 orthogonal projections of A on DE and DF respectively.Define B_1, B_2, C_1, C_2 cyclically. Euler lines of $AA_1A_2, BB_1B_2, CC_1C_2$ concur at $X(11263)$.*

Theorem 2.3. *Let $I = X(1)$ be incenter of ABC and DEF circumcevian triangle of I. A_1, A_2 orthogonal projections of A on DE and DF respectively. Define B_1, B_2, C_1, C_2 cyclically. Euler lines of $DA_1A_2, EB_1B_2, FC_1C_2$ concur at $X(10)$-Spieker center of ABC (Figure 2).*

Theorem 2.4. *Let $I = X(1)$-incenter of ABC and DEF circumcevian triangle of I. A_1, A_2 orthogonal projections of I on AE and AF respectively. Define B_1B_2, C_1, C_2 cyclically. Euler lines of $IA_1A_2, IB_1B_2, IC_1C_2$ concur. Concurrency point is $X(1319)$=BEVAN-SCHRÖDER POINT (Figure 3).*

Theorem 2.5. *Let $I = X(1)$-incenter of ABC and DEF circumcevian triangle of I. A_1, A_2 orthogonal projections of I on AE and AF respectively. Define B_1B_2, C_1, C_2 cyclically. Euler lines of $AA_1A_2, BB_1B_2, CC_1C_2$ concur. Concurrency point is $X(2646)$=SUM OF PU(80) (Figure 4).*

Theorem 2.6. *Let P be a point on Euler of ABC and DEF circumcevian triangle of P. A_1, A_2 orthogonal projections of P on DB and DC respectively. Define B_1B_2, C_1, C_2 cyclically. Euler lines of $ABC, PA_1A2, PB_1B_2, PC_1C_2$ concur. (Figure 5).*

Let $Q(P)$ be concurrency point of four Euler lines. Some pairs $(P, Q(P)) = (X_i, X_j) = (2, 7426), (3, 30), (4, 403), (5, 10096), (20, 16386)$.

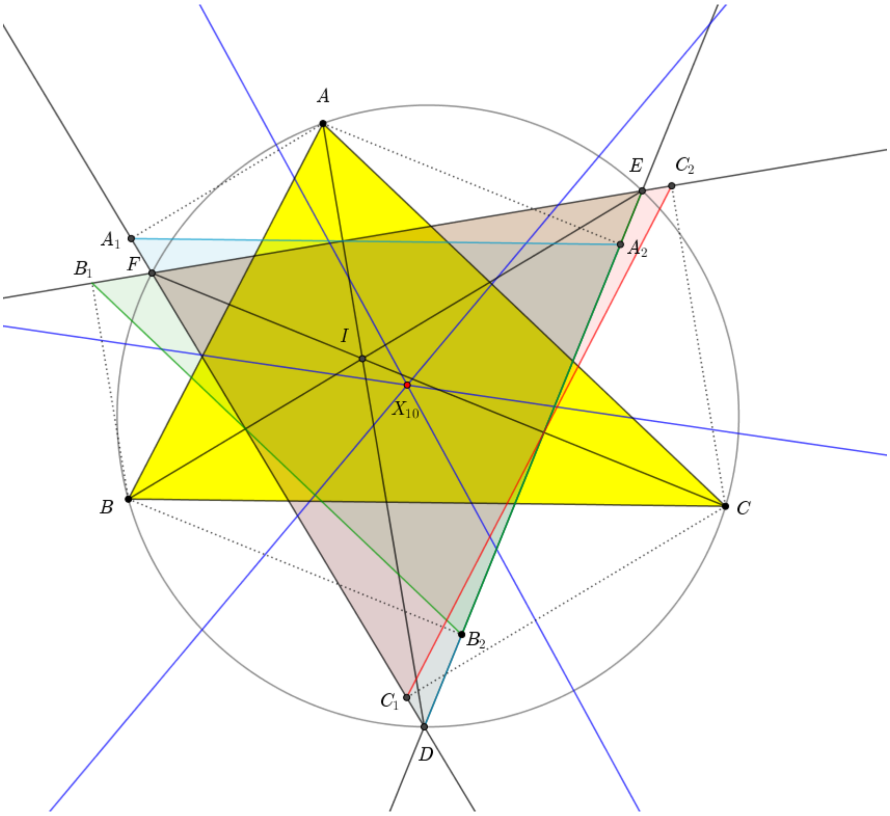

FIGURE 2. Spieker Center As Concurrent Euler Lines

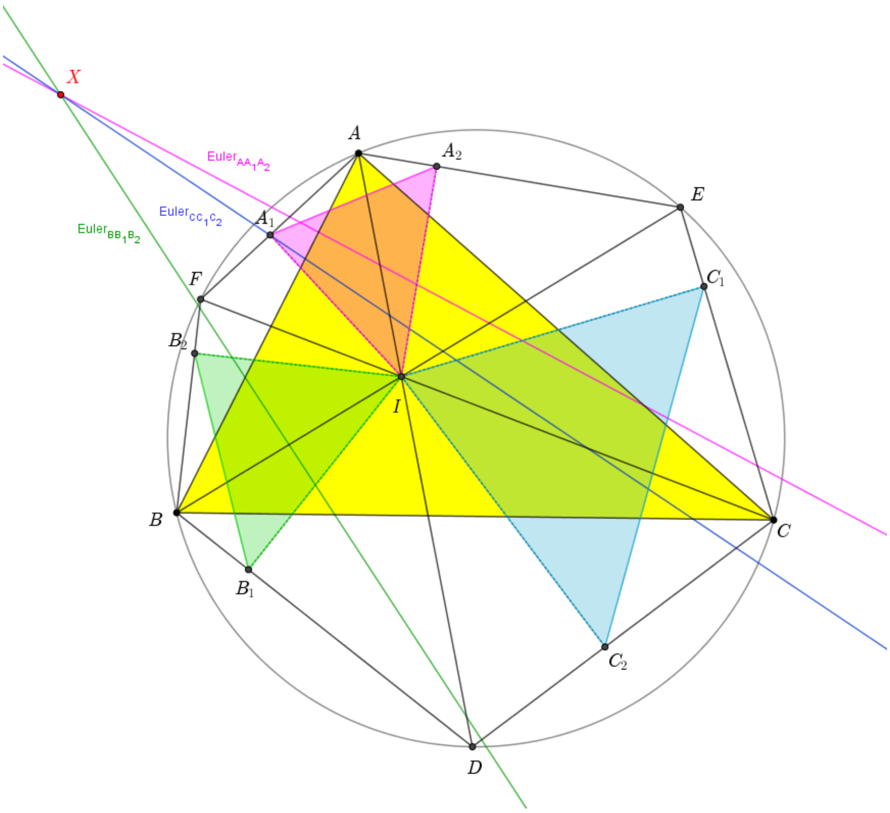

FIGURE 3.

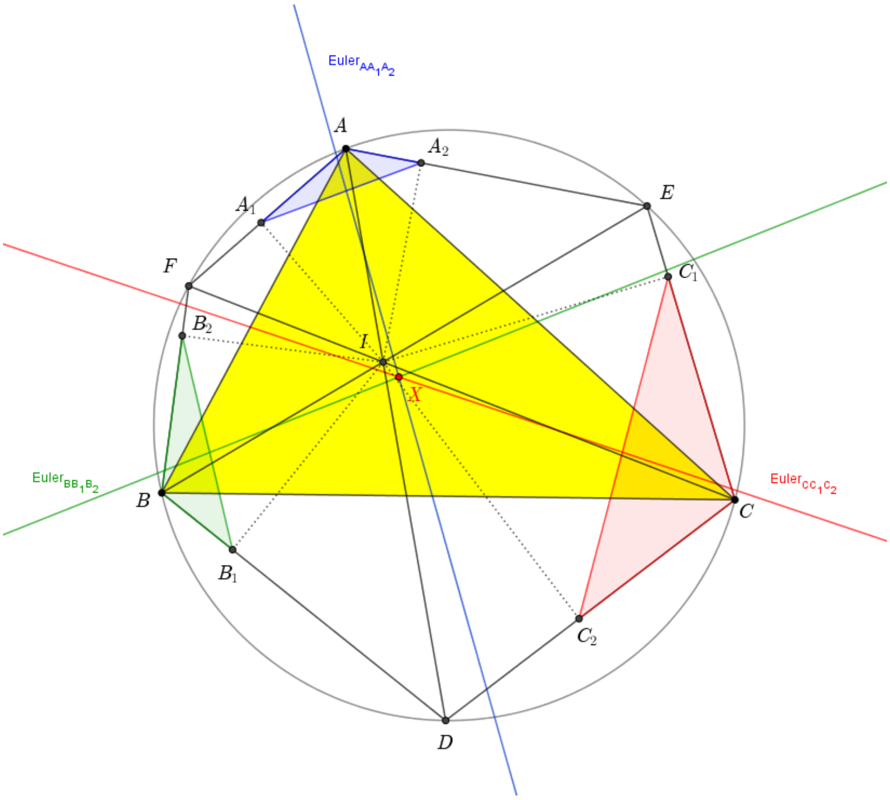

FIGURE 4.

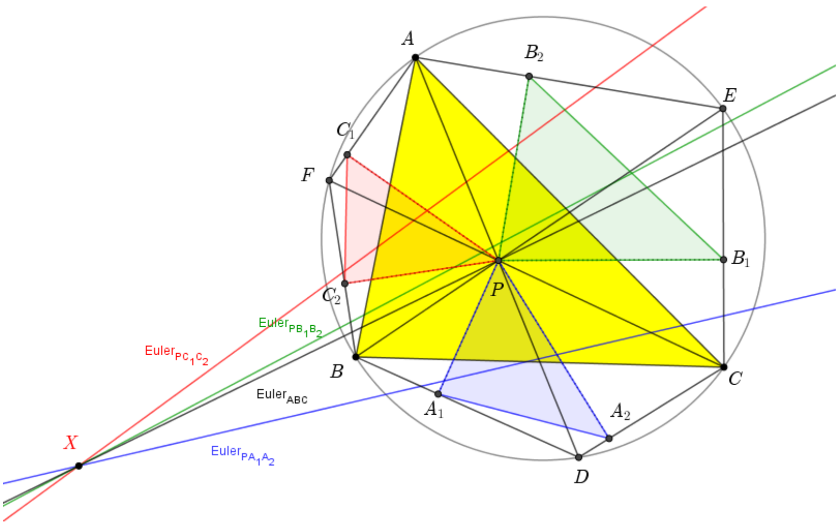

FIGURE 5.

Theorem 2.7. *Let P be a point on Euler of ABC and DEF circumcevian triangle of P. A_1, A_2 orthogonal projections of P on DB and DC respectively. Define B_1B_2, C_1, C_2 cyclically. Euler lines of $ABC, DA_1A2, EB_1B_2, FC_1C_2$ concur (Figure 6).*

For $P = X(2)$-centroid of ABC, concurrency point has first barycentric coordinates: $(4a^6 - a^4b^2 - 4a^2b^4 + b^6 - a^4c^2 - 6a^2b^2c^2 - b^4c^2 - 4a^2c^4 - b^2c^4 + c^6 ::)$ on Euler line of ABC.

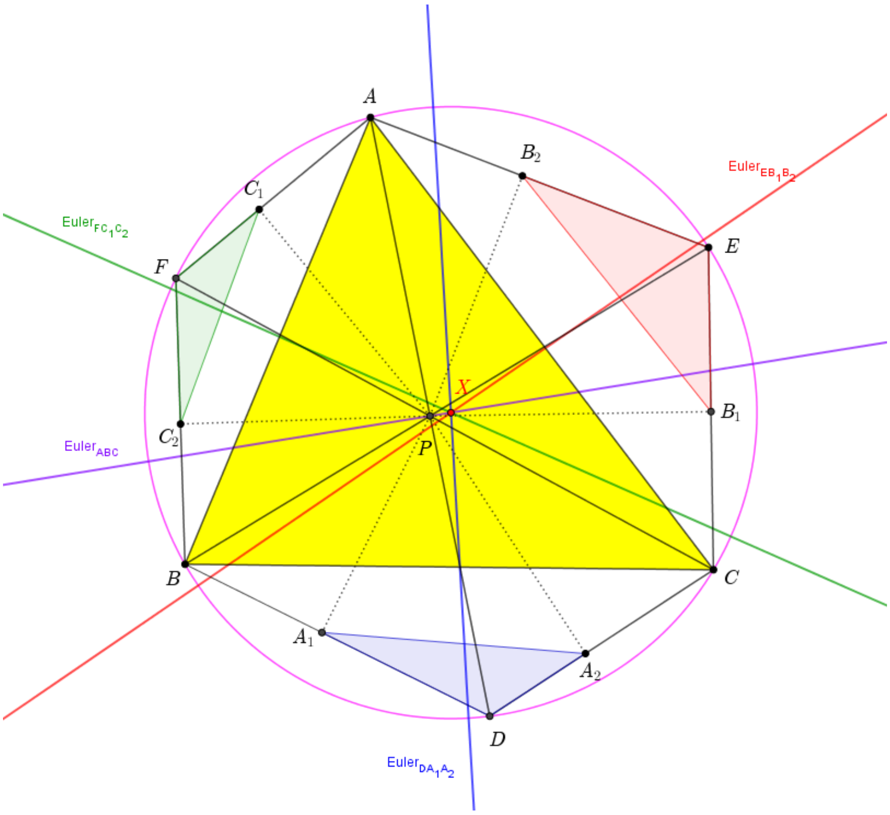

FIGURE 6.

For $P = X(20)$-DeLongchamp's point of ABC, concurrency point has first barycentric coordinates: $(-4a^{10} + 7a^8(b^2 + c^2) + (b^2 - c^2)^4(b^2 + c^2) + 2a^6(b^4 - 17b^2c^2 + c^4) + 2a^2(b^2 - c^2)^2(b^4 + 7b^2c^2 + c^4) - 4a^4(2b^6 - 5b^4c^2 - 5b^2c^4 + 2c^6) ::)$ on Euler line of ABC.

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