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Intangents triangle

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Abstract. We present problems for Students about triangles similar (but not homothetic) or homothetic with the Intangents triangle. The problems are discovered by the computer program "Discoverer", created by the authors.

Keywords. Intangents triangle, triangle geometry, Euclidean geometry, computer discovered mathematics, "Discoverer".

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1. Introduction

Given triangle ABC, there are four lines simultaneously tangent to the incircle and the A-excircle. Of these, three correspond to the sidelines of the triangle, and the fourth is known as the A-intangent. The intangents intersect one another pairwise, and their points of intersection form the so-called Intangents triangle. See Intangents triangle in [6], [1]..

The computer program "Discoverer" created by the authors, [3], [4], has discovered many theorems about triangles similar (but not homothetic) or homothetic with the Intangents triangle. Here we present a few of these theorems as problems for students.

Given triangle ABC, we denote the side lengths as follows: $a = BC, b = CA$ and $c = AB$.

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2. Similar Triangles

References for Problem 1: Pedal triangle and Inversion in [6].

Problem 1. *The Intangents triangle is similar (but not homothetic) to the Pedal Triangle of the Inverse of the Orthocenter in the Circumcircle. The ratio of similarity is*

$$k = \frac{|(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)|}{2(b+c-a)(c+a-b)(a+b-c)\sqrt{E}},$$

where

$$E = a^6+b^6+c^6+3a^2b^2c^2-a^2b^4-c^4b^2-b^4c^2-c^4a^2-a^4c^2-a^4b^2.$$

References for Problem 2: Circumcevian triangle and Inversion in [6].

Problem 2. *The Intangents triangle is similar (but not homothetic) to the Circumcevian Triangle of the Inverse of the Orthocenter in the Circumcircle. The ratio of similarity is*

$$k = \frac{|(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)|}{abc(b+c-a)(c+a-b)(a+b-c)},$$

References for Problem 3: Nine-Point Center in [6]

Problem 3. *Denote by O the Circumcenter of triangle ABC, and denote by Qa the Nine-Point Center of triangle OBC, Qb the Nine-Point Center of triangle OCA, and Qc the Nine-Point Center of triangle OAB. Then the Intangents triangle is similar (but not homothetic) to triangle $QaQbQc$ (known as the Triangle of the Nine-Point Centers of the Triangulation Triangles of the Circumcenter). The ratio of similarity is*

$$k = \frac{\sqrt{E}}{2(b+c-a)(c+a-b)(a+b-c)},$$

where E is as in Problem 1.

References for Problem 4: Pedal triangle and Inversion in [6]

Problem 4. *Denote by P the Inverse of the Orthocenter in the Circumcircle. Denote by $HaHbHc$ the Pedal triangle of P and by Qa the Orthocenter of triangle $AHbHc$, Qb the Orthocenter of triangle $HaBHc$, and Qc the Orthocenter of triangle $HaHbC$. Then the Intangents triangle is similar (but not homothetic) to triangle $QaQbQc$ (known as the Triangle of the Orthocenters of the Pedal Corner Triangles of the Inverse of the Orthocenter in the Circumcircle). The ratio of similarity is*

$$\frac{|(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)|}{2(b+c-a)(c+a-b)(a+b-c)\sqrt{E}},$$

where E is as in Problem 1.

References for Problem 5: Pedal triangle, Nine-Point Center and Inversion in [6]

Problem 5. *Denote by P the Inverse of the Nine-Point Center in the Circumcircle. Denote by $HaHbHc$ the Pedal triangle of P and by Qa the Nine-Point Center of triangle $AHbHc$, Qb the Nine-Point Center of triangle $HaBHc$, and Qc the Nine-Point Center of triangle $HaHbC$. Then the Intangents triangle is* *similar (but not homothetic) to triangle QaQbQc (known as the Triangle of the Nine-Point Centers of the Pedal Corner Triangles of the Inverse of the Nine-Point Center in the Circumcircle). The ratio of similarity is*

$$\frac{|(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)|}{2(b+c-a)(c+a-b)(a+b-c)\sqrt{E}},$$

where E is as in Problem 1 (The same as in Problem 4).

References for Problem 6: Pedal triangle and Far-Out Point in [6]

Problem 6. *Denote by $HaHbHc$ the Pedal triangle of the Far-Out Point and by Qa the Centroid of triangle $AHbHc$, Qb the Centroid of triangle $HaBHc$, and Qc the Centroid of triangle $HaHbC$. Then the Intangents triangle is similar (but not homothetic) to triangle QaQbQc (known as the Triangle of the Centroids of the Pedal Corner Triangles of the Far-Out Point). The ratio of similarity is*

$$\frac{|(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)|}{2(b+c-a)(c+a-b)(a+b-c)\sqrt{E}},$$

where E is as in Problem 1 (The same as in Problem 4).

3. Homothetic Triangles

Reference for Problem 7: Orthic triangle in [6].

Problem 7. *The Intangents triangle is homothetic to the Orthic triangle. The ratio of homothety is*

$$k = \frac{-(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{2abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is obtuse, then $k > 0$.
If triangle ABC is acute, then $k < 0$.

References for Problem 8: Tangential triangle in [6], Internal center of similitude if two circles in [6].

Problem 8. *The Intangents triangle is homothetic to the Tangential triangle. The center of homothety is the Internal Center of Similitude of Circumcircle and Incircle. The ratio of homothety is*

$$k = \frac{-2abc}{(b+c-a)(c+a-b)(a+b-c)} < 0.$$

References for Problem 9: Kosnita triangle in [5].

Problem 9. *The Intangents triangle is homothetic to the Kosnita triangle. The ratio of homothety is*

$$k = \frac{-abc}{(b+c-a)(c+a-b)(a+b-c)} < 0.$$

References for Problem 10: Symmedian point and Tangential triangle in [6].

Problem 10. *Denote by K the Symmedian point. Denote by $TaTbTc$ the Tangential triangle, and by Qa the midpoint of segment KTa, Qb the midpoint of segment KTb, and Qc the midpoint of segment KTc. Then the Intangents triangle is homothetic to triangle $QaQbQc$ (known as the Euler Anticevian Triangle of the Symmedian Point). The ratio of the homothety is*

$$k = \frac{-abc}{(b+c-a)(c+a-b)(a+b-c)} < 0.$$

References for Problem 11: Symmedian point, Cevian triangle and Antimedial triangle in [6], Retrocenter = Symmedian point of the Antimedial triangle.

Problem 11. *Denote by R the Retrocenter. Denote by $RaRbRc$ the Cevian triangle of the Retrocenter and by Qa the midpoint of segment ARa, Qb the midpoint of segment BRb, and Qc the midpoint of segment CRc. Then the Intangents triangle is homothetic to triangle $QaQbQc$ (known as the Half-Cevian Triangle of the Retrocenter). The ratio of the homothety is*

$$k = \frac{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{4abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

Reference for Problem 12: Circum-Orthic triangle in [6].

Problem 12. *The Intangents triangle is homothetic to the Circum-Orthic Triangle. The ratio of the homothety is*

$$k = \frac{-(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k < 0$.
If triangle ABC is obtuse, then $k > 0$.

Reference for Problem 13: Circum-Anticevian Triangle in [2].

Problem 13. *The Intangents triangle is homothetic to the Circum-Anticevian Triangle of the Centroid. The ratio of the homothety is*

$$k = \frac{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

Problem 14. *The Intangents triangle is homothetic to the Triangle of Reflections of the Circumcenter in the Sidelines of the Medal Triangle. The ratio of the homothety is*

$$k = \frac{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{2abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

Reference for Problem 15: de Longchamps point in [6].

Problem 15. *The Intangents triangle is homothetic to the Triangle of Reflections of the de Longchamps point in the Sidelines of the Antimedial Triangle. The ratio of the homothety is*

$$k = \frac{2(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

Problem 16. *Denote by $HaHbHc$ the Orthic triangle of triangle ABC and by Qa the Orthocenter of triangle $AHbHc$, Qb the Orthocenter of triangle $HaBHc$, Qc the Orthocenter of triangle $HaHbC$. The Intangents triangle is homothetic to triangle $QaQbQc$ (known as the Triangle of the Orthocenters of the Cevian Corner Triangles of the Orthocenter). The ratio of the homothety is*

$$k = \frac{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{2abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

References for Problem 17: Nine-Poin Center and Orthic triangle in [6].

Problem 17. *The Intangents triangle is homothetic to the Triangle of Reflections of the Nine-Point Center in the Sidelines of the Orthic triangle. The ratio of the homothety is*

$$k = \frac{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2-c^2)}{2abc(b+c-a)(c+a-b)(a+b-c)}.$$

If triangle ABC is acute, then $k > 0$.
If triangle ABC is obtuse, then $k < 0$.

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