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Extended Soddy Configurations

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Abstract. By using the computer programs "Geometry Expressions" and "Wolfram Mathematica", we give theorems about extended Soddy circles configuration.

Keywords. Soddy Circles, Soddy Line, Outer and Inner cousin soddy circles, computer-discovered mathematics, Euclidean geometry, Geometry Expressions.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

In the general Apollonius problem [1] it is known that, given three arbitrary circles with noncollinear centers, there are at most 8 circles tangent to each of them. In the special case when three given circles are tangent externally to each other, there are only two such circles. These are called the inner and outer Soddy circles respectively [2].

Frederick Soddy (1936) gave the formula for finding the radii of the Soddy circles given the radii r_i $(i = 1, 2, 3)$ of the other three.[5]

The centers are called the inner S_1 and outer Soddy centers S_2 respectively. The triangle line that passes through the inner and outer Soddy centers S_1 and S_2 is called Soddy line of triangle ABC. [3]. The inner Soddy center is the equal detour point X_{176} (Kimberling 1994) [6]. (Figure 1)

Given a triangle ABC with inner and outer Soddy centers S_1 and S_2, respectively, the inner Soddy triangle PQR (respectively, outer Soddy triangle $P'Q'R'$ is the triangle formed by the points of tangency of the inner (respectively, outer) Soddy circle with the three mutually tangent circles centered at each of the vertices of ABC [7].

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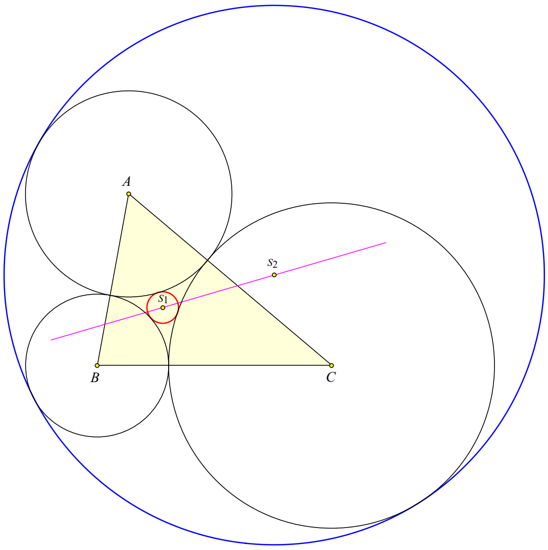

FIGURE 1. Soddy Circles and Soddy Line

These triangles were explicitly mentioned but not named by Oldknow (1996). The name "Soddy triangles" is therefore proposed for the first time in [7].

See also Dao Thanh Oai [8].

2. THEOREMS ON EXTENDED SODDY CONFIGURATIONS

Construct three circles internally tangent to outer sodyy circle and externally tangent to three circles with centers M_a, M_b, M_c. These circles are called extended soddy circles of outer soddy circle.

Theorem 2.1. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let (M_a), (M_b), (M_c) be the extended soddy circles of outer soddy circle. (Figure 2). The lines AM_a, BM_b, CM_c concur on the Soddy line S_1S_2 at point X_{481}, first Eppstein point of $\triangle ABC$, whose barycentric coordinates are*

$$X_{481}\left((a+b-c)(a-b+c)(a^2-a(b+c)+2S),:\cdots:\cdots\right)$$

Theorem 2.2. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let (M_a), (M_b), (M_c) be the extended soddy circles of outer soddy circle (Figure 3), let T_{aa}, T_{bb}, T_{cc} be the touch points of (S_2) with (M_a), (M_b), (M_c) respectively. The lines AT_{aa}, BT_{bb}, CT_{cc} concur on the Soddy line S_1S_2 at point X whose barycentric coordinates are*

$$X\left(5a^3+5a^2(b+c)-5(b-c)^2(b+c)-5a(b-c)^2-12aS):\cdots:\cdots\right)$$

where S is twice the area of $\triangle ABC$.

Theorem 2.3. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let $\triangle DEF$ the pedal triangle of incenter, let (M_a), (M_b),*

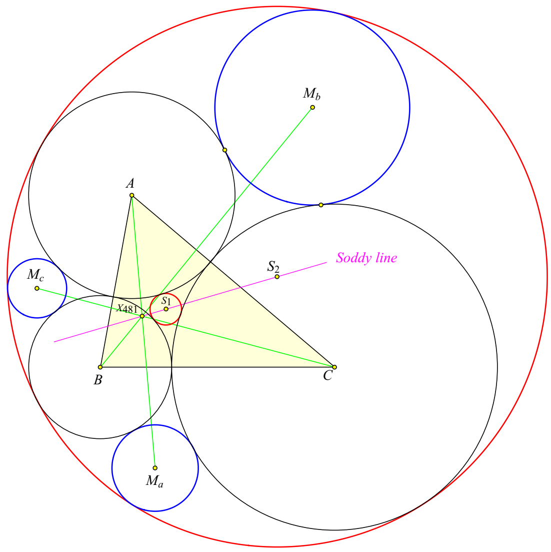

FIGURE 2.

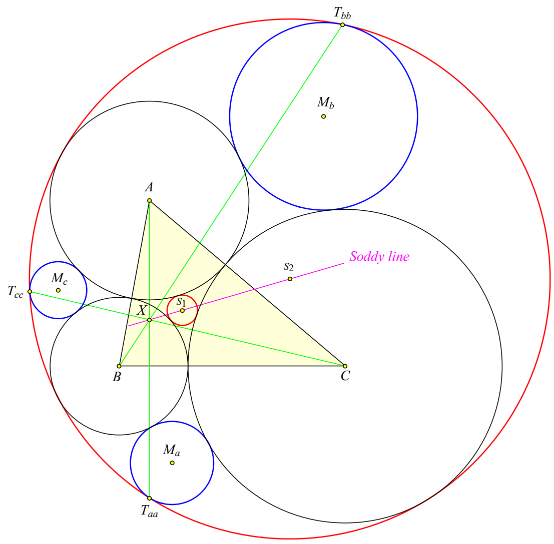

FIGURE 3.

(M_c) be the extended soddy circles of outer soddy circle. The lines DM_a, EM_b, FM_c concur on the Soddy line S_1S_2 at point X whose barycentric coordinates are

$$X\left((a^2-(b-c)^2)(2a^2-2a(b+c)+S:\cdots:\cdots)\right)$$

where S is twice the area of $\triangle ABC$. (Figure 4)

Theorem 2.4. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let (M_a), (M_b), (M_c) be the extended soddy circles of outer soddy circle, let T_{ab}, T_{ac} be the touch points of (M_a) with (B), (C) respectively; the points T_{ba}, T_{bc} and T_{ca}, T_{cb}, are defined similarly (Figure 5). The six points T_{ab}, T_{ac}, T_{ba}, T_{bc}, T_{ca}, T_{cb} lie on a circle (called outer soddy cousin circle) centered at point X which lies on Soddy line S_1S_2. The barycentric coordinates of X are*

$$X\left(a^3+a^2(b+c)-(b-c)^2(b+c)-a(b-c)^2-5aS):\cdots:\cdots\right)$$

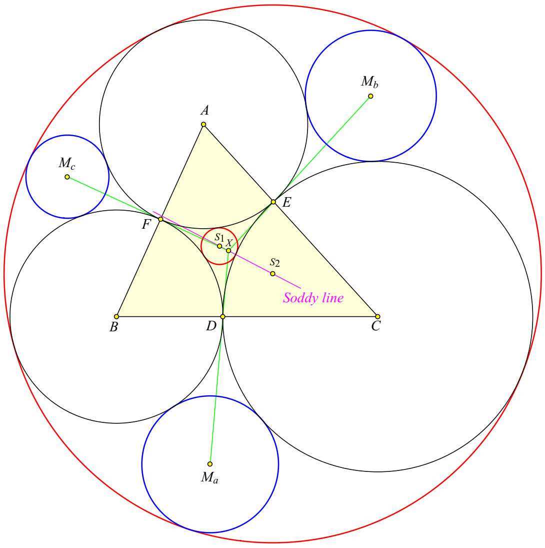

FIGURE 4.

and

$$\frac{S_1X}{XS_2} = -\frac{9\,[4p^2 + (r + 4R)\,(r + 4R - 4p)]}{(r + 4R - 2p)(r + 4R + 2p)}$$

where S, R, r, p *are twice area, circumradius, inradius and semiperimeter of* $\triangle ABC$ *respectively.*

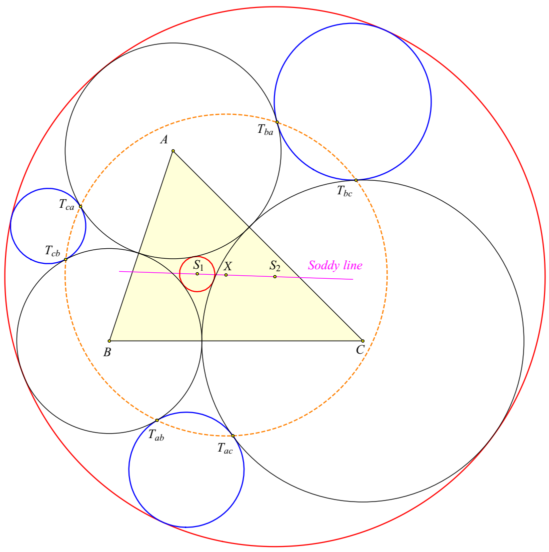

FIGURE 5.

Construct three circles externally tangent to inner sodyy circle and internally tangent to three circles with centers N_a, N_b, N_c. These circles are called extended soddy circles of inner soddy circle.

Theorem 2.5. *Let* $\triangle ABC$ *be a triangle, let* (S_1) *and* (S_2) *be the inner and outer Soddy circles respectively, let* (N_a), (N_b), (N_c) *be the inner extended soddy circles of inner soddy circle (Figure 6), let* S_{aa}, S_{bb}, S_{cc} *be the touch points of* (S_2) *with*

(N_a), (N_b), (N_c) *respectively. The lines* AS_{aa}, BS_{bb}, CS_{cc} *concur on the Soddy line* S_1S_2 *at point* X *whose barycentric coordinates are*

$$X\left(5a^3+5a^2(b+c)-5(b-c)^2(b+c)-5a(b-c)^2+12aS) : \cdots : \cdots\right)$$

where S *is twice the area of* $\triangle ABC$.

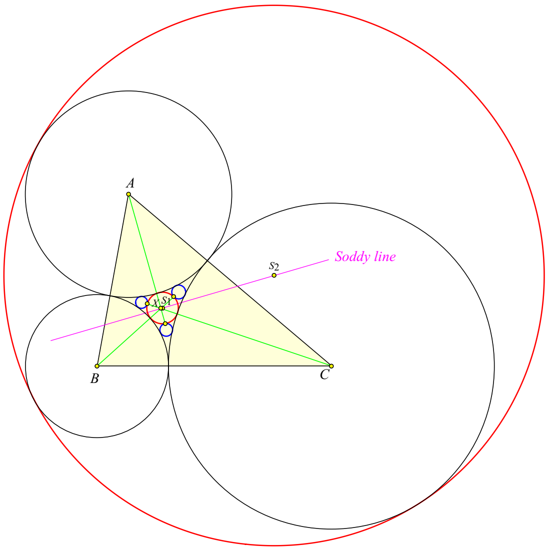

FIGURE 6.

Theorem 2.6. *Let* $\triangle ABC$ *be a triangle, let* (S_1) *and* (S_2) *be the inner and outer Soddy circles respectively, let* (N_a), (N_b), (N_c) *be the extended soddy circles of inner soddy circle (Figure 7). The lines* AN_a, BN_b, CN_c *concur on the Soddy line* S_1S_2 *at point* X_{482}, *second Eppstein point of* $\triangle ABC$, *whose barycentric coordinates are*

$$X_{482}\left((a+b-c)(a-b+c)(a^2-a(b+c)-2S), : \cdots : \cdots\right)$$

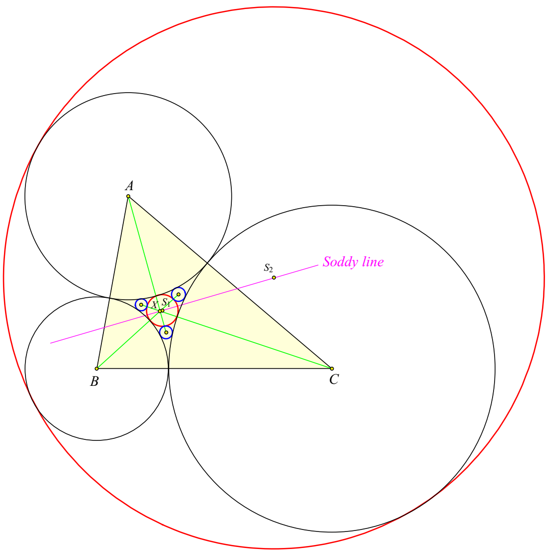

FIGURE 7.

Theorem 2.7. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let $\triangle DEF$ the pedal triangle of incenter, let (N_a), (N_b), (N_c) be the extended soddy circles of inner soddy circle (Figure 8). The lines DN_a, EN_b, FN_c concur on the Soddy line S_1S_2 at point X whose barycentric coordinates are*

$$X\left((a^2-(b-c)^2)(2a^2-2a(b+c)-S):\cdots:\cdots)\right)$$

where S is twice the area of $\triangle ABC$.

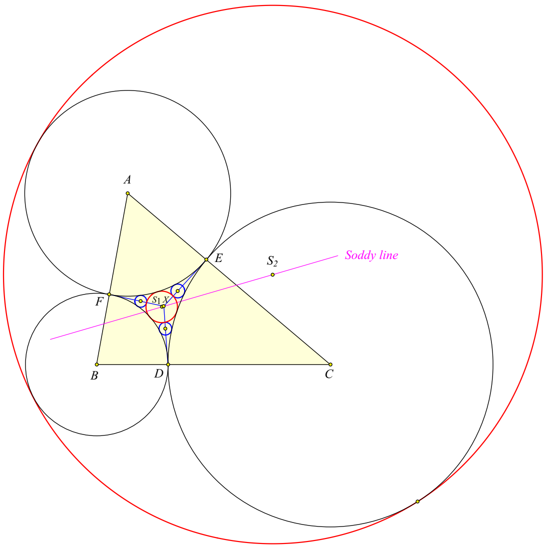

FIGURE 8.

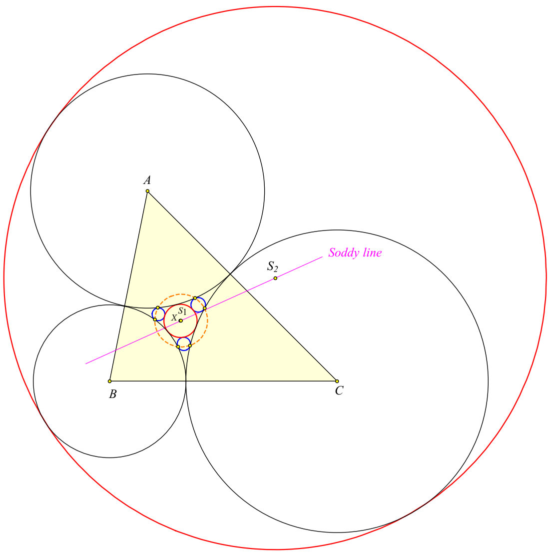

FIGURE 9.

Theorem 2.8. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let (N_a), (N_b), (N_c) be the extended soddy circles of inner soddy circle, let S_{ab}, S_{ac} be the touch points of (N_a) with (B), (C) respectively; the points S_{ba}, S_{bc} and S_{ca}, S_{cb}, are defined similarly (Figure9). The six points S_{ab}, S_{ac}, S_{ba}, S_{bc}, S_{ca}, S_{cb} lie on a circle (called Inner Soddy Cousin Circle)*

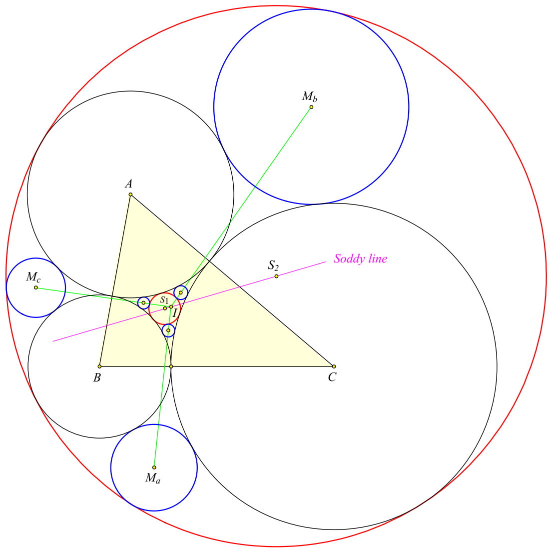

FIGURE 10.

centered at point X which lies on Soddy line S_1S_2. The barycentric coordinates of X are

$$X\left(a^3+a^2(b+c)-(b-c)^2(b+c)-a(b-c)^2+5aS\right):\cdots:\cdots)$$

and

$$\frac{S_1X}{XS_2}=\frac{S-r(r+4R)}{9\left(r^2+4rR+S\right)}$$

where S, R, r, p are twice area, circumradius, inradius and semiperimeter of $\triangle ABC$ respectively.

Theorem 2.9. *Let $\triangle ABC$ be a triangle, let (S_1) and (S_2) be the inner and outer Soddy circles respectively, let (M_a), (M_b), (M_c) be the extended soddy circles of outer soddy circle, let (N_a), (N_b), (N_c) be the extended soddy circles of inner soddy circle (Figure 10). The lines M_aN_a, M_bN_b, M_cN_c concur on the Soddy line S_1S_2 at the incenter I of $\triangle ABC$.*

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