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The relation between three concurrent diagonals of a hexagon and rectangles attached to sides of a triangle

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Abstract. The article gives some new results on three concurrent diagonals of a hexagon related to rectangles attached to sides of a given triangle.

Keywords. Three concurrent diagonals of a hexagon, rectangles attached to sides of a triangle.

1. INTRODUCTION

The following is an interesting theorem in a mathematical book [2]:

Theorem 1.1. *Given a triangle ABC. Three squares ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let the points of intersection of AA_b with CC_b and BB_a be N and P respectively; CC_a with BB_a and AA_c be Q, R respectively; BB_c with AA_c and CC_b be S, M, respectively. Prove that three lines MQ, NR, PS are concurrent at a point.*

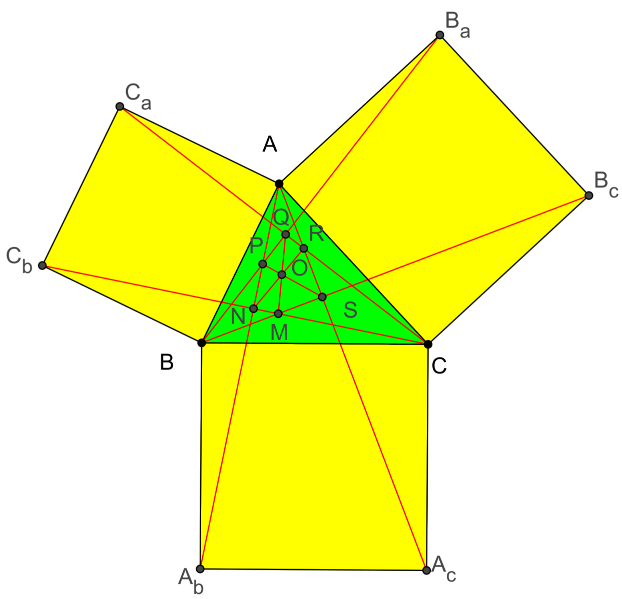

Figure 1

[h]

Since this theorem, we refer to the following one:

Theorem 1.2. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let the points of intersection of AA_b with CC_b and BB_a be N and P, respectively; CC_a with BB_a and AA_c be Q, R, respectively; BB_c with AA_c and CC_b be S, M, respectively. Prove that three lines MQ, NR, PS are concurrent at a point.*

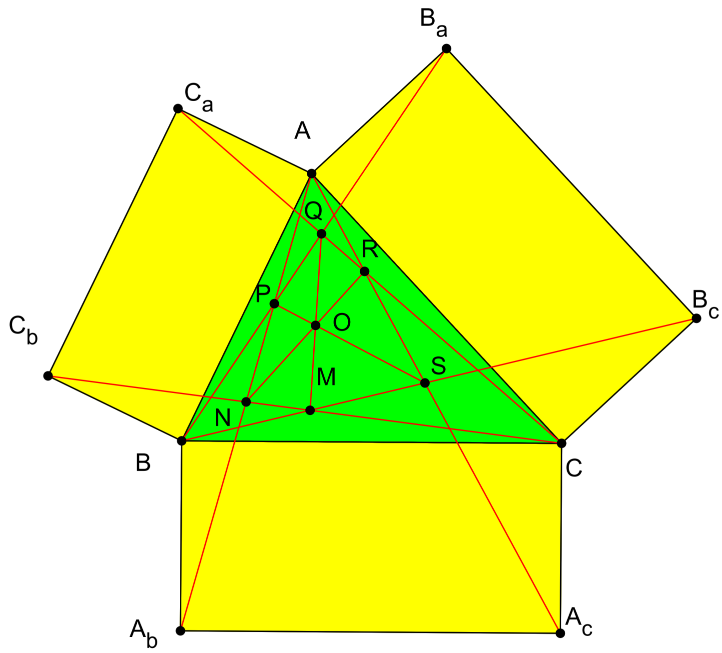

Figure 2

2. CONTENTS

The following are some new theorems.

Theorem 2.1. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let A_1, A_2; B_1, B_2; C_1, C_2 be points on A_bA_c, B_cB_a, C_aC_b such that $A_bA_1 = A_1A_2 = A_2A_c$; $B_cB_1 = B_1B_2 = B_2B_a$; $C_aC_1 = C_1C_2 = C_2B$, respectively. Let the points of intersection of AA_1 with CC_2 and BB_2 be N and P, respectively, CC_1 with BB_2 and AA_2 be Q, R, respectively; BB_1 with AA_2 and CC_2 be S, M, respectively. Prove that three lines MQ, NR, PS are concurrent at a point.*

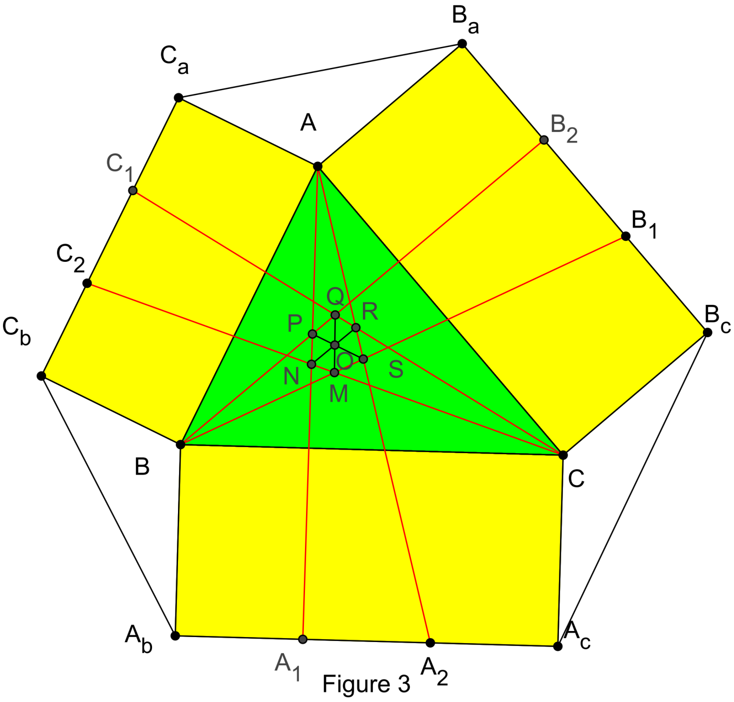

Figure 3

Theorem 2.2. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let A', B', C' be the midpoints of segments B_aC_a, C_bA_b, A_cB_c, respectively. Let the points of intersection of AB' with $A'B$ be C''; $A'C$ with AC' be B''; BC' with $B'C$ be A''. Prove that AA'', BB'', CC'' are concurrent at a point.*

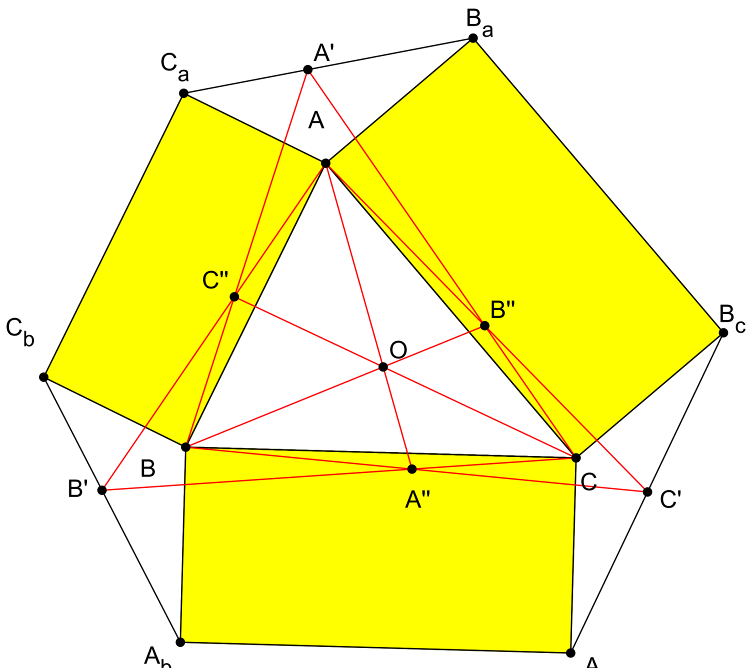

Figure 4

Theorem 2.3. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let A', B', C' be the in-centers of triangles AB_aC_a, BC_bA_b, CA_cB_c, respectively. Let the points of intersection of AB' with $A'B$ be C''; $A'C$ with AC' be B''; BC' with $B'C$ be A". Prove that AA'', BB'', CC'' are concurrent at a point.*

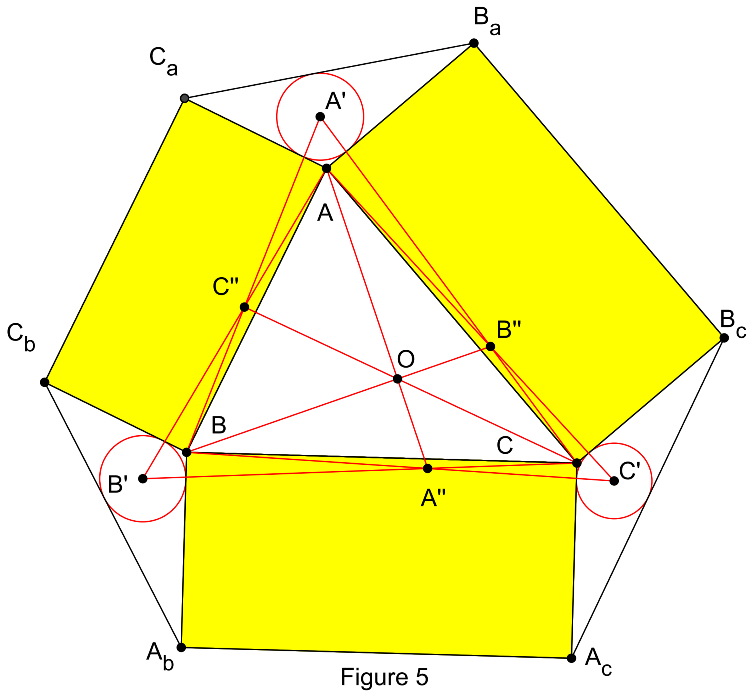

Figure 5

Theorem 2.4. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let A', B', C' be the ex-centers of triangles AB_aC_a, BC_bA_b, CA_cB_c, respectively.*

Let the points of intersection of AB' with $A'B$ be C'; $A'C$ with AC' be B''; BC' with $B'C$ be A''. Prove that AA'', BB'', CC'' are concurrent at a point.

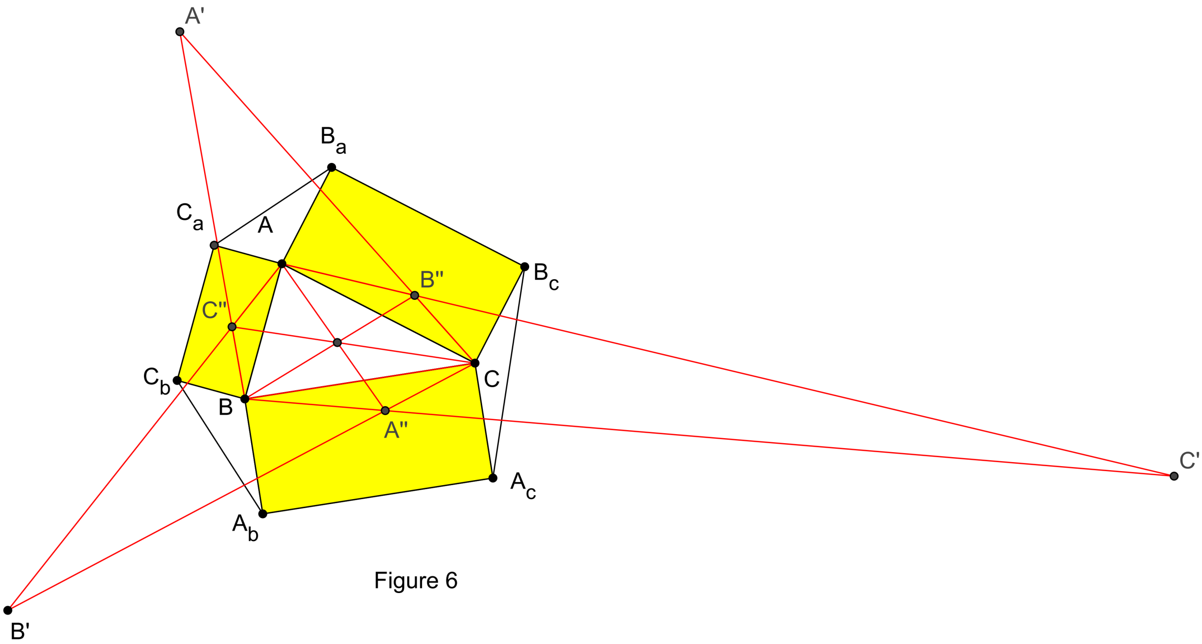

Figure 6

Theorem 2.5. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. H_a, H_b, H_c are the feet of the altitudes dropped from A, B, C to B_aC_a, C_bA_b, A_cB_c, respectively. Let A', B', C' be the points on AH_a, BH_b, CH_c such that $\frac{AA'}{AH_a} = \frac{BB'}{BH_b} = \frac{CC'}{CH_c} = k$, respectively. $BA' \cap AB' = C''$; $BC' \cap CB' = C''$; $CA' \cap AC' = B''$. Prove that AA'', BB'', CC'' are concurrent at a point.*

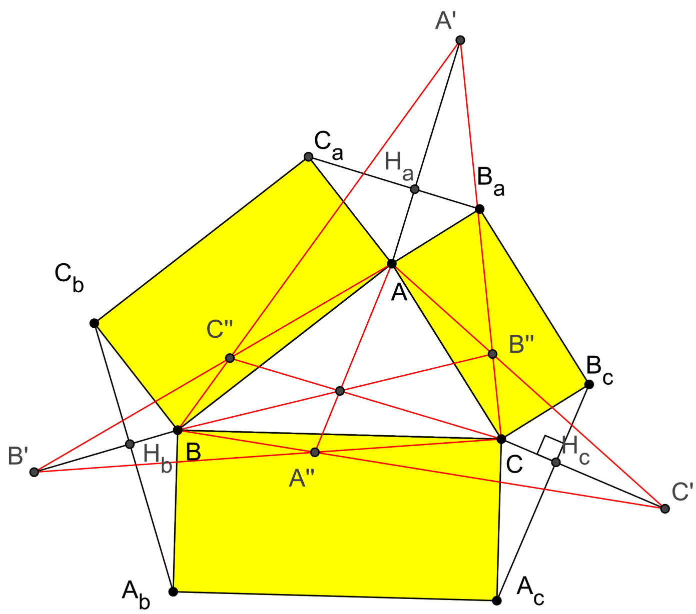

Figure 7

Theorem 2.6. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let M, N, P be the midpoints of segments B_aC_a, C_bA_b, A_cB_c. Let A', B', C' be the points on AM, BN, CP such that $\frac{AA'}{AM} = \frac{BB'}{BN} = \frac{CC'}{CP} = k$, respectively. $BA' \cap AB' = C''$; $BC' \cap CB' = C''$; $CA' \cap AC' = B''$. Prove that AA'', BB'', CC'' are concurrent at a point.*

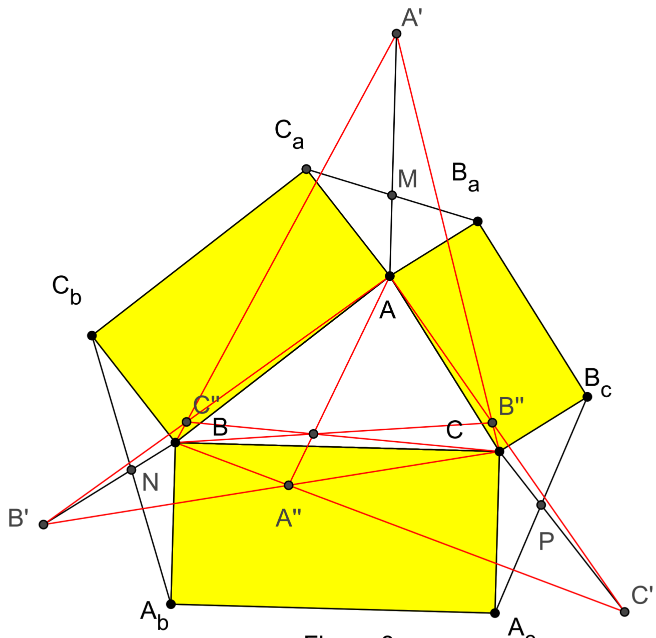

Figure 8

Theorem 2.7. *Given a triangle ABC. Three similar rectangles ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on the three sides have the same outer orientation. Let M, N, P be the circumcenters of triangles AB_aC_a, BC_bA_b, CA_cB_c, respectively. Let A', B', C' be the points on AM, BN, CP such that $\frac{AA'}{AM} = \frac{BB'}{BN} = \frac{CC'}{CP} = k$, respectively. $BA' \cap AB' = C''$; $BC' \cap CB' = C''$; $CA' \cap AC' = B''$. Prove that AA'', BB'', CC'' are concurrent at a point.*

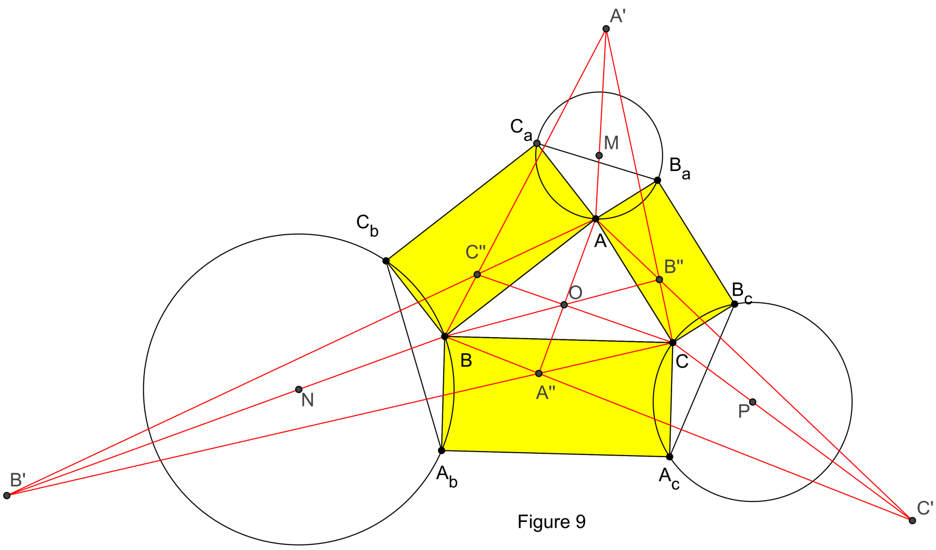

Figure 9

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[2] Nguyen Minh Ha (2015), *The oriented geometry, The Dan Tri publishing house.*
[3] Vietnamese Forum: *Nice problem, nice solutions, the passion of mathematics*