International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 2, 2017, pp.86-89
Received 10 March 2017. Published on-line 10 May 2017
web: http://www.journal-1.eu/

Triangles Homothetic With Triangle ABC

Sava Grozdev[a], Hiroshi Okumura[b] and Deko Dekov[c] [2]
[a] VUZF University of Finance, Business and Entrepreneurship,
Gusla Street 1, 1618 Sofia, Bulgaria
e-mail: sava.grozdev@gmail.com
[b] Department of Mathematics, Yamato University, Osaka, Japan
e-mail: okumura.hiroshi@yamato-u.ac.jp
[c]Zahari Knjazheski 81, 6000 Stara Zagora, Bulgaria
e-mail: ddekov@ddekov.eu
web: http://www.ddekov.eu/

Abstract. We study triangles homothetic with Triangle ABC.

Keywords. triangle geometry, remarkable point, computer-discovered mathematics, Euclidean geometry.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

The homothety is a powerful tool for investigation of geometrical objects. Here we will accent on construction of barycentric coordinates of notable points of triangle T, provided T is the homothetic image of the reference triangle ABC under a given homothety with center O and ratios k.

We use barycentric coordinates [1]-[15]. The Kimberling points are denoted by X(n) ([8]). Given triangle ABC, the side lengths are denoted $a = BC, b = CA, c = AB$. The are of ABC is denoted by Δ.

[2]Corresponding author

1. General Case

Given a homothety $h(O,k)$ with center $O = (p,q,r)$ and ratio k.

Theorem 1.1. *The barycentric coordinates of the homothetic image hP of a point $P = (u,v,w)$ wrt the homothety $h(O,k)$ are as follows:*

$$\begin{aligned} hP = (&up + pv + pw + kuq + kur - kpv - kpw, \\ &uq + vq + qw + kpv + kvr - kuq - kqw, \\ &ur + vr + wr + kpw + kqw - kur - kvr). \end{aligned}$$

Proof. We use the homothety formula (17), [5]. □

Denote by $T(O,k) = TaTbTc$ the triangle which is the homothetic image of triangle ABC wrt homothety $h(O,k)$.

Theorem 1.2. *The barycentric coordinates of triangle $T(O,k)$ are as follows:*

$$\begin{aligned} Ta &= (p + kq + kr, -q(-1+k), -r(-1+k)), \\ Tb &= (-p(-1+k), q + kp + kr, -r(-1+k)), \\ Tc &= (-p(-1+k), -q(-1+k), r + kp + kq), \end{aligned}$$

2. Special Cases

Table 1 gives a few homotheties from triangle ABC to triangle T:

	Triangle T	**Center**	**Ratio**
1	Medial Triangle	X(2)	-1/2
2	Antimedial Triangle	X(2)	-2
3	Euler Triangle	X(4)	1/2
4	Aquila Triangle	X(1)	2
5	Johnson Triangle	X(5)	-1
6	Circumcevian triangle of the Circumcenter	X(3)	-1
7	Half-Median Triangle	X(2)	1/4
8	First Stanilov Triangle Triangle	X(2)	-4/5
9	Second Stanilov Triangle	X(2)	4
10	Inner Yff Triangle	X(1)	k > 0
11	Outer Yff Triangle	X(1)	k < 0
12	Inner Grebe Triangle	X(6)	k < 0
13	Outer Grebe Triangle	X(6)	k > 0

Table 1.

Theorem 2.1. *Triangle ABC is homothetic with the Inner Yff triangle. The Center of the homothety is X(1) the Incenter. The ratio of the homothety is*

$$k = \frac{(b+c-a)(c+a-b)(a+b-c)}{a^2b + a^2c + ab^2 + b^2c + c^2a + c^2b - a^3 - b^3 - c^3} > 0.$$

Theorem 2.2. *Triangle ABC is homothetic with the Outer Yff triangle. The Center of the homothety is X(1) the Incenter. The ratio of the homothety is*

$$k = \frac{(b+c-a)(c+a-b)(a+b-c)}{a^2b+a^2c+ab^2+b^2c+c^2a+c^2b-4abc-a^3-b^3-c^3} < 0.$$

Theorem 2.3. *Triangle ABC is homothetic with the Inner Grebe triangle. The Center of the homothety is X(6) the Symmedian Point. The ratio of the homothety is*

$$k = \frac{4\Delta - 2(a^2+b^2+c^2)}{4\Delta} < 0.$$

Theorem 2.4. *Triangle ABC is homothetic with the Outer Grebe triangle. The Center of the homothety is X(6) the Symmedian Point. The ratio of the homothety is*

$$k = \frac{4\Delta + 2(a^2+b^2+c^2)}{4\Delta} > 0.$$

By using the corresponding homothety, we can easily find the barycentric coordinates of the triangle-image of ABC under the homothety.

Example 2.1. *The barycentric coordinates of the Medial triangle are as follows:*

$$Ta = (0,1,1), \quad Tb = (1,0,1), \quad Tc = (1,1,0).$$

Below we give an example of calculations of barycentric coordinates of a new notable point. The selected new point is not available in [8].

Example 2.2. *The barycentric coordinates of the Clawson Point of the Medial Triangle $P = (u,v,w)$ are as follows:*

$$u = (b+c)(b^2+c^2-a^2)(a^2+b^2+c^2-2bc),$$
$$v = (c+a)(c^2+a^2-b^2)(b^2+c^2+a^2-2ca),$$
$$w = (a+b)(a^2+b^2-c^2)(c^2+a^2+b^2-2ab).$$

Problem 2.1. *Find the barycentric coordinates of the following new notable points of the Inner Grebe triangle: Nagel point, Spieker center, Feuerbach point.*

Problem 2.2. *Find the barycentric coordinates of the following new notable points of the Outer Grebe triangle: Nagel point, Spieker center, Feuerbach point.*

REFERENCES

[1] César Lozada, Index of triangles referenced in ETC. `http://faculty.evansville.edu/ck6/encyclopedia/IndexOfTrianglesReferencedInETC.html`.

[2] Francisco Javier García Capitán, *Barycentric Coordinates*, International Journal of Computer Discovered Mathematics, 2015, vol.0, no.0, 32-48. `http://www.journal-1.eu/2015/01/Francisco-Javier-Barycentric-Coordinates-pp.32-48.pdf`.

[3] Pierre Douillet, *Translation of the Kimberling's Glossary into barycentrics*, 2012, v48, `http://www.douillet.info/~douillet/triangle/Glossary.pdf`.

[4] S. Grozdev and D. Dekov, *A Survey of Mathematics Discovered by Computers*, International Journal of Computer Discovered Mathematics, 2015, vol.0, no.0, 3-20. `http://www.journal-1.eu/2015/01/Grozdev-Dekov-A-Survey-pp.3-20.pdf`.

[5] S. Grozdev and D. Dekov, Barycentric Coordinates: Formula Sheet, International Journal of Computer Discovered Mathematics, vol.1, 2016, no 2, 75-82. `http://www.journal-1.eu/2016-2/Grozdev-Dekov-Barycentric-Coordinates-pp.75-82.pdf`.

[6] S. Grozdev and V. Nenkov, *Three Remarkable Points on the Medians of a Triangle* (Bulgarian), Sofia, Archimedes, 2012.

[7] S. Grozdev and V. Nenkov, *On the Orthocenter in the Plane and in the Space* (Bulgarian), Sofia, Archimedes, 2012.

[8] C. Kimberling, *Encyclopedia of Triangle Centers - ETC*, http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.

[9] Gerry Leversha, *The Geometry of the Triangle*, The United Kingdom Mathematical Trust, The Pathways Series no.2, 2013.

[10] G. Paskalev and I. Tchobanov, *Remarkable Points in the Triangle* (in Bulgarian), Sofia, Narodna Prosveta, 1985.

[11] G. Paskalev, *With coordinates in Geometry* (in Bulgarian), Sofia, Modul-96, 2000.

[12] M. Schindler and K.Cheny, Barycentric Coordinates in Olympiad Geometry, 2012, http://www.mit.edu/~evanchen/handouts/bary/bary-full.pdf.

[13] E. W. Weisstein, *MathWorld - A Wolfram Web Resource*, http://mathworld.wolfram.com/.

[14] *Wikipedia*, https://en.wikipedia.org/wiki/.

[15] P. Yiu, *Introduction to the Geometry of the Triangle*, 2001, new version of 2013, http://math.fau.edu/Yiu/YIUIntroductionToTriangleGeometry130411.pdf.

[16] P. Yiu, *The uses of homogeneous barycentric coordinates in plane euclidean geometry*, Int. J. Math. Educ. Sci. Technol. 2000, vol.31, 569-578.