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Computer Discovered Mathematics: Problems about Points on the Euler line

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Abstract. By using the computer program "Discoverer" we present problems about points on the Euler line of a triangle.

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Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

In this note we present a list of 342 new notable points of a triangle which lie on the Euler line of the triangle. (see the Supplementary material) The presented points are not available in the Kimberling's ETC [11].

We encourage the reader to prove that the points lie on the Euler line and to find their barycentric coordinates. We encourage the readers to publish the results in our journal.

The problems are discovered by the computer program "Discoverer", created by the authors.

Recall that the Euler line of a triangle is the line connecting the Centroid and the Circumcenter of the triangle.

We use barycentric coordinates. See [1] - [19]. We denote by $a = BC, b = CA$ and $c = AB$ the side lengths of a triangle ABC. The barycentric coordinates of the form $(f(a,b,c), f(b,c,a), f(c,a,b))$ are shortened to $[f(a,b,c)]$

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Note that we can prove that a point P in barycentric coordinates lies on the Euler line, if we use the barycentric equation of the Euler line L:

$L : (-c^2+b^2)(b^2+c^2-a^2)x+(-a^2+c^2)(c^2+a^2-b^2)y+(-b^2+a^2)(a^2+b^2-c^2)z = 0.$

Below is the first problem of the enclosed list.

Problem 1. *Prove that the Center of the Tangential Circle of the Medial Triangle lies on the Euler line. Find the barycentric coordinates of this point.*

Figure 1 illustrates Problem 1. In figure 1,

- G is the Centroid,
- O is the Circumcenter,
- GO is the Euler line,
- $MaMbMc$ is the Medial triangle,
- P is the center of Tangential circle, and
- Q is the complement of point P, that is, Q is the Center of tangential circle of the Medial triangle.

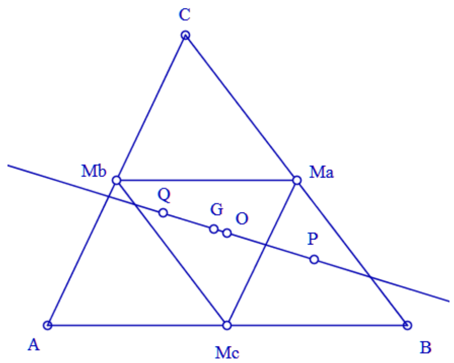

FIGURE 1.

Solution. We will use the construction of a complement of a point in a triangle. Recall that a complement of a point is the homothetic image of the point under the homothety with center the centroid of the triangle and ratio $-\frac{1}{2}$. If a point P has barycentric coordinates (u, v, w), then its complement has barycentric coordinates $(v + w, w + u, u + v)$.

Given triangle ABC. Denote by P the Center of the Tangential circle (this is point X(26) in [11]), and by Q its complement, that is, the Center of the Tangential circle of the Medial triangle. Since point P lies on the Euler line (see e.g. [12]), point Q also lies on the Euler line. The barycentric coordinates of these points are as follows:

$$P = \left[a^2(a^8 - 2a^6(b^2 + c^2) - (b^2 - c^2)^2(b^4 + c^4) + 2a^2(b^6 + c^6))\right],$$
$$Q = \left[u = b^2\left(b^8 - 2b^6(c^2 + a^2) - (c^2 - a^2)^2(c^4 + a^4) + 2b^2(c^6 + a^6)\right)\right]$$
$$+c^2\left(c^8 - 2c^6(a^2 + b^2) - (a^2 - b^2)^2(a^4 + b^4) + 2c^2(a^6 + b^6)\right)].$$

Below we give two theorems about the new point.

Theorem 1. *The Center of the Tangential circle of the Medial triangle is the Center of the homothety of the Half-Median Triangle and the Triangle of the Circumcenters of the Pedal Corner Triangles of the Center of the Tangential Circle.*

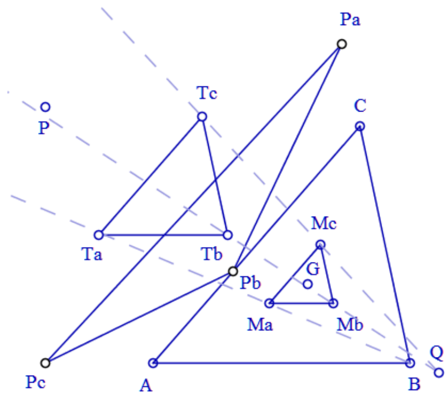

Figure 2.

Figure 2 illustrates Theorem 1. In figure 2,

- G is the Centroid,
- $MaMbMc$ is the Half-median triangle,
- P is the Center of the Tangential circle,
- $PaPbPc$ is the Pedal triangle of P,
- Ta is the Circumcenter of triangle $APbPc$,
- Tb is the Circumcenter of triangle $BPcPa$,
- Tc is the Circumcenter of triangle $CPaPb$,
- $TaTbTc$ is the Triangle of the Circumcenters of the Pedal Corner Triangles of P, and
- Q is the Center of the Tangential circle of the Medial triangle.

Then triangles $MaMbMc$ and $TaTbTc$ are homothetic and point Q is the center of the homothety.

Problem 1a. Find the ratio of the homothety in the above theorem.

Theorem 2. *The Center of the Tangential circle of the Medial triangle is the Homothetic Center of the Triangle ABC and the Triangle of the Orthocenters of the Antipedal Corner Triangles of the Center of the Tangential Circle.*

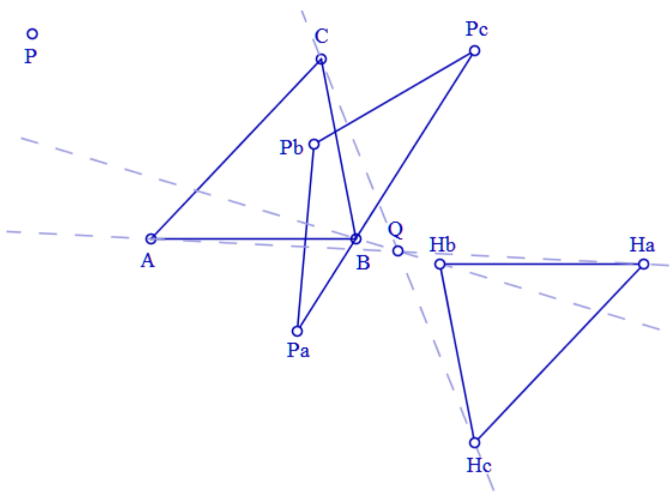

Figure 3.

Figure 3 illustrates Theorem 2. In figure 3,

- P is the Center of the Tangential Circle,
- $PaPbPc$ is the Antipedal triangle of P,
- Ha is the Orthocenter of triangle $PaBC$,
- Hb is the Orthocenter of triangle $PbCA$,
- Hc is the Orthocenter of triangle $PcAB$,
- $HaHbHc$ is the Triangle of the Orthocenters of the Antipedal Corner Triangles of P, and
- Q is the Center of the Tangential circle of the Medial triangle.

Then triangles ABC and $HaHbHc$ are homothetic and point Q is the center of the homothety.

Problem 1b. Find the ratio of the homothety in the above theorem.

SUPPLEMENTARY MATERIAL

The enclosed supplementary material contains problems related to the topic.

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