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Computer Discovered Mathematics: Haimov Triangle of the Incenter

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Abstract. By using the computer program "Discoverer", we present theorems about Haimov Triangle of the Incenter.

Keywords. Haimov Triangle of the Incenter, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, Discoverer.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

The computer program "Discoverer", created by Grozdev and Dekov, with collaboration by Hiroshi Okumura, is the first computer program, able easily to discover new theorems in mathematics, and the first computer program, able easily to discover new knowledge in science. See [3].

In 2015 Haim Haimov [7] has introduced a triangle now known as the *Haimov triangle of point P*. By using the "Discoverer" Grozdev and Dekov [4] have investigated the Haimov triange of a point P and the special case when P is the Centroid. In this paper, by using the "Discoverer", we investigate the Haimov triangle of the Incenter. We expect that the majority of the theorems are new, discovered by a computer.

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2. Haimov Triangle of the Incenter

Recall the definition of the Haimov triangle of the Incenter. See [7]. Let I be the Incenter of $\triangle ABC$. Let $A_1B_1C_1$ be the Cevian triangle of I, that is, the Incentral Triangle of $\triangle ABC$. Points A, B and B_1 define a circle, and points A, C and C_1 define another circle. Point A is an intersection of these two circles. Label by Qa the second intersection of the circles. Similarly, define points Qb and Qc. Then $\triangle QaQbQc$ is the *Haimov triangle of the Incenter*.

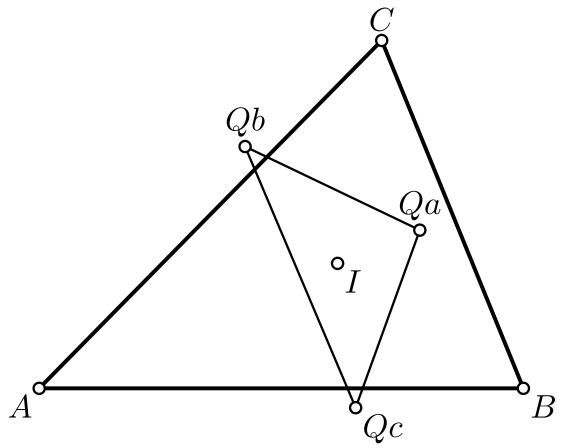

Figure 1.

Figure 1 illustrates the definition of the Haimov triangle of the Incenter. In figure 1, I is the Incenter and $QaQbQc$ is the Haimov Triangle of the Incenter.

From theorem 3.1, [4] we deduce:

Theorem 2.1. *The barycentric coordinates of the Haimov triangle of the Incenter $QaQbQc$ are as follows:*

$$Qa = (a(b^2 + c^2 - a^2 - bc), b^2(a + b), c^2(a + c)),$$

$$Qb = (a^2(a + b), (b(a^2 + c^2 - b^2 - ac), c^2(b + c)),$$

$$Qc = (a^2(a + c), b^2(b + c), (c(a^2 + b^2 - c^2 - ab)).$$

3. Preliminaries

In this section we review some basic facts about barycentric coordinates. We refer the reader to [5],[6],[10],[9],[14],[1].

The labeling of triangle centers follows Kimberling's ETC [8]. For example, X(1) denotes the Incenter, X(2) denotes the Centroid, X(37) is the Grinberg Point, X(75) is the Moses point (Note that in the prototype of the "Discoverer" the Moses point is the X(35)), etc.

The reader may find definitions in [12],[13],[2].

The reference triangle ABC has vertices $A = (1, 0, 0), B(0, 1, 0)$ and C(0,0,1). The side lengths of $\triangle ABC$ are denoted by $a = BC, b = CA$ and $c = AB$. A point is an element of $\mathbb{R}^3$, defined up to a proportionality factor, that is,

$$\text{For all } k \in \mathbb{R} - \{0\} : \; P = (u, v, w) \text{ means that } P = (u, v, w) = (ku, kv, kw).$$

The equation of the line joining two points with coordinates u_1, v_1, w_1 and u_2, v_2, w_2 is

$$\begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ x & y & z \end{vmatrix} = 0. \tag{3.1}$$

Three lines $p_i x + q_i y + r_i z = 0$, $i = 1, 2, 3$ are concurrent if and only if

$$\begin{vmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{vmatrix} = 0 \tag{3.2}$$

The intersection of two lines $L_1 : p_1x + q_1y + r_1z = 0$ and $L_2 : p_2x + q_2y + r_2z = 0$ is the point

$$(q_1r_2 - q_2r_1, r_1p_2 - r_2p_1, p_1q_2 - p_2q_1) \tag{3.3}$$

Given a point $P(u, v, w)$, the complement of P is the point $(v + w, w + u, u + v)$, the anticomplement of P is the point $(-u + v + w, -v + w + u, -w + u + v)$, the isotomic conjugate of P is the point (vw, wu, uv), and the isogonal conjugate of P is the point (a^2vw, b^2wu, c^2uv).

4. Theorems

We give a few theorems about triangles perspective with the Haimov triangle of the Incenter. These theorems are discovered by the "Discoverer". Theorem 4.1 below is a corollary to theorem 3.2 in [4], and a part of theorem 4.2 is a corollary to Theorem 3.4 in [4].

Theorem 4.1. *The Perspector of the Haimov Triangle of the Incenter and Triangle ABC is the X(58) Isogonal Conjugate of the Spieker Center.*

Theorem 4.2. *The Perspector of the Haimov Triangle of the Incenter and the Incentral Triangle is the X(214) Complement of the Isogonal Conjugate of the Inverse of the Incenter in the Circumcircle.*

Theorem 4.3. *The Perspector of the Haimov Triangle of the Incenter and the Cevian Triangle of the External Center of Similitude of the Incircle and the Circumcircle is the X(36) Inverse of the Incenter in the Circumcircle.*

Theorem 4.4. *The Perspector of the Haimov Triangle of the Incenter and the Circumcevian Triangle of the Isogonal Conjugate of the Spieker Center is the X(58) Isogonal Conjugate of the Spieker Center.*

Theorem 4.5. *The Perspector of the Haimov Triangle of the Incenter and the Triangle of Reflections of the Spieker Center in the Sidelines of the Anticevian Triangle of the Incenter is the X(58) Isogonal Conjugate of the Spieker Center.*

Theorem 4.6. *The Perspector of the Haimov Triangle of the Incenter and the Euler Triangle of the Isogonal Conjugate of the Spieker Center is the X(58) Isogonal Conjugate of the Spieker Center.*

Theorem 4.7. *The Perspector of the Haimov Triangle of the Incenter and the Anticevian Euler Triangle of the Isogonal Conjugate of the Spieker Center is the X(58) Isogonal Conjugate of the Spieker Center.*

5. Proofs

We will prove theorem 4.3.

Proof of Theorem 4.3. The barycentric coordinates of the Haimov Triangle of the Incenter are given in theorem 2.1. The barycentric coordinates of the External Center of Similitude of the Incircle and the Circumcircle are as follows [8]: $P = (\frac{a^2}{-a+b+c}, \frac{b^2}{-b+c+a}, \frac{c^2}{-c+a+b})$. The cevian triangle $PaPbPc$ of P has barycentric coordinates $Pa = (0, Pb, Pc)$, $Pb = (Pa, 0, Pc)$ and $Pc = (Pa, Pb, 0)$.

By using (3.1), we find the equation of the line $L_1 = PaQa$:

$$b^2c^2(a+b+c)(b-c)x + ac^2(b^2+c^2-a^2-bc)(-b+c+a)y$$
$$-ab^2(b^2+c^2-a^2-bc)(-c+a+b)z = 0.$$

Similarly, we find the equations of the lines $L_2 = PbQb$ and $L_3 = PcQc$.

By using (3.2) we conclude that the lines L_1, L_2 and L_3 concur in a point. Then by using (3.3) we find the intersection point of lines L_1 and L_2. The intersection point has barycentric coordinates

$$(a^2(b^2+c^2-a^2-bc), b^2(c^2+a^2-b^2-ca), c^2(a^2+b^2-c^2-ab))$$

This is the point X(36), the Inverse of the Incenter in the Circumcircle. □

6. Constructions of the Haimov Triangle of the Incenter

We can use Theorems 4.1 to 4.7 in order to construct by straightedge and compass [11] the Haimov triangle of the Incenter, denoted by $\triangle Q_aQ_bQ_c$.

If we take theorems 4.1 and 4.2, the construction is as follows: Construct the Isogonal Conjugate of the Spieker Center and label it S. Construct the Complement of the Isogonal Conjugate of the Inverse of the Incenter in the Circumcircle and label it R. Construct the Incentral triangle $PaPbPc$. Then Qa is the point of intersection of lines AS and RPa. Analogously, Qb is the point of intersection of lines BS and RPb and Qc is the point of intersection of lines CS and RPc. Figure 2 illustrates the construction.

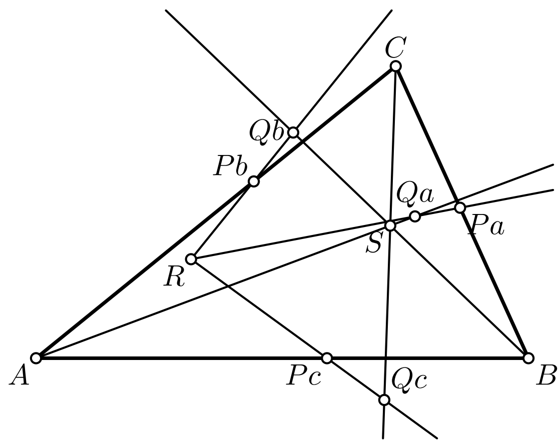

FIGURE 2.

We can use the set of theorems 4.1 to 4.7 in order to find additional constructions of the Haimov triangle of the Incenter.

Also, we could use the above constructions in order to find in a relatively simple way the barycentric coordinates of the Haimov triangle of the Incenter.

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