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Computer Discovered Mathematics: Stanilov Triangles

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Abstract. By using the computer program "Discoverer" we study the Stanilov First and Second Triangles.

Keywords. Stanilov Triangles, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, "Discoverer".

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1. Introduction

In 2004, Grozyo Stanilov gave the description of two triangles [15], now known as the *Stanilov triangles*.

In this note we use the computer program "Discoverer" [5] in order to study the Staniliv triangles. Note that a number of results about Stanilov triangles are given in [1],[2].

The description of the triangles is as follows:

The First Stanilov triangle is the homothetic image of triangle ABC under the homothety with center the Centroid of triangle ABC and ratio $-\frac{4}{5}$. See Figure 1.

The Second Stanilov triangle is the homothetic image of triangle ABC under the homothety with center the Centroid of triangle ABC and ratio 4.

We use barycentric coordinates. We refer the reader to [6],[7],[8],[11],[12],[18],[4], [3],[14],[13],[10],[9].

The reader could use the Supplementary material in order to create new problems.

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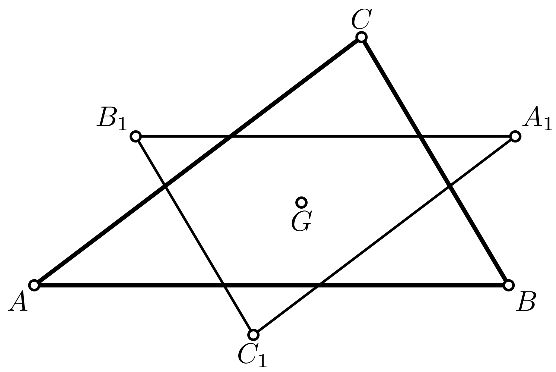

Figure 1.

Theorem 1.1. *Given a homothety with center the Centroid and the ratio k. Then the homothetic image of triangle ABC,*

$$DEF = h(G,k)(ABC)$$

has the following barycentric coordinates:

$$D = (1+2k, 1-k, 1-k), \quad E = (1-k, 1+2k, 1-k), \quad F = (1-k, 1-k, 1+2k).$$

Proof. We use formula [6, §5, (17)] □

2. First Stanilov Triangle

2.1. **Barycentric Coordinates.** From Theorem 1.1 we obtain:

Theorem 2.1. *The barycentric coordinates of the First Stanilov Triangle $A_1B_1C_1$ are as follows:*

$$A_1 = (-1, 3, 3), \quad B_1 = (3, -1, 3), \quad C_1 = (3, 3, -1).$$

2.2. **Notable Points.** The computer program "Discoverer" has investigated 201 notable points of the First Stanilov Triangle. Of these 11 are available in [9] and the rest of 190 notable points are new notable points. See the Supplementary Material, Folder 1.

Table 1 gives notable points of the First Stanilov Triangle in terms of the notable points of the Reference triangle ABC that are Kimberling notable points $X(n)$. Denote by T the First Stanilov Triangle of Triangle ABC.

	Notable Point of T	Notable Point of Triangle ABC
1	X(1) Incenter	X(3617)
2	X(2) Centroid	X(2)
3	X(3) Circumcenter	X(3091)
4	X(4) Orthocenter	X(3522)

Table 1.

2.3. **Internal Center of Similitude.** The computer program "Discoverer" has investigated 841 Internal Similitude Centers of Circles of the First Stanilov Triangle. Of these 45 are available in [9] and the rest of 797 notable points are new notable points. See the Supplementary Material, Folder 2. For example:

Theorem 2.2. *The Internal Center of Similitude of the Circumcircle of Triangle ABC and the Nine-Point Circle of the First Stanilov Triangle is the X(3523).*

Theorem 2.3. *The Internal Center of Similitude of the Antimedial Circle of Triangle ABC and the Circumcircle of the First Stanilov Triangle is the X(3832).*

2.4. **External Center of Similitude.** The computer program "Discoverer" has investigated 841 External Similitude Centers of Circles of the Second Stanilov Triangle. Of these 37 are available in [9] and the rest of 804 notable points are new notable points. See the Supplementary Material, Folder 3. For example:

Theorem 2.4. *The External Center of Similitude of the Circumcircle of Triangle ABC and the Circumcircle of the First Stanilov Triangle is the X(3146).*

Theorem 2.5. *The External Center of Similitude of the Antimedial Circle of Triangle ABC and the Nine-Point Circle of the First Stanilov Triangle is the X(3).*

3. SECOND STANILOV TRIANGLE

3.1. **Barycentric Coordinates.** From Theorem 1.1 we obtain:

Theorem 3.1. *The barycentric coordinates of the Second Stanilov Triangle $A_2B_2C_2$ are as follows:*

$$A_2 = (3, -1, -1), \quad B_2 = (-1, 3, -1), \quad C_2 = (-1, -1, 3).$$

3.2. **Notable Points.** The computer program "Discoverer" has investigated 201 notable points. Of these 33 are available in [9] and the rest of 190 notable points are new notable points. See the Supplementary Material, Folder 4.

Table 2 gives the classical notable points of the Second Stanilov Triangle in terms of the notable points of the Reference Triangle ABC that are Kimberling notable points $X(n)$. Denote by T the Second Stanilov Triangle of Triangle ABC.

	Notable Point of T	Notable Point of Triangle ABC
1	X(1) Incenter	X(145)
2	X(2) Centroid	X(2)
3	X(3) Circumcenter	X(20)
4	X(4) Orthocenter	X(3146)

TABLE 2.

3.3. **Internal Center of Similitude.** The computer program "Discoverer" has investigated 840 Internal Similitude Centers of Circles of the Second Stanilov Triangle. Of these 49 are available in [9] and the rest of 791 notable points are new notable points. See the Supplementary Material, Folder 5. For example:

Theorem 3.2. *The Internal Center of Similitude of the Circumcircle of Triangle ABC and the Circumcircle of the Second Stanilov Triangle is the X(3522).*

Theorem 3.3. *The Internal Center of Similitude of the Antimedial Circle of Triangle ABC and the Spieker Circle of the Second Stanilov Triangle is the X(3434).*

3.4. **External Center of Similitude.** The computer program "Discoverer" has investigated 839 External Similitude Centers of Circles of the Second Stanilov Triangle. Of these 69 are available in [9] and the rest of 772 notable points are new notable points. See the Supplementary Material, Folder 6. For example:

Theorem 3.4. *The External Center of Similitude of the Circumcircle of Triangle ABC and the Spieker Circle of the Second Stanilov Triangle is the X(100).*

Theorem 3.5. *The External Center of Similitude of the Orthocentroidal Circle of Triangle ABC and the Nine-Point Circle of the Second Stanilov Triangle is the X(2553).*

Supplementary Material

The enclosed supplementary material contains theorems related to the topic.

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