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The Nine Circles Problem and the Sixteen Points Circle

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Abstract. In this note we introduce a problem of nine circles related with the Pascal theorem, and the Brianchon theorem, and problem 3845 in Crux Mathematicorum. We present a problem of 16 points circle associated with the Bundle theorem configuration.

Keywords. Pascal theorem, Brianchon theorem, Bundle theorem, Möbius plane, Euclidean plane.

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1. The Nine Circles Problem

Theorem 1 ([1]). *Consider the following configuration:*
Points $A_1, A_2, A_3, A_4, A_5, A_6$ *lie on a circle,*
points $B_1, B_2, B_3, B_4, B_5, B_6$ *lie on a circle,*
points A_1, A_2, B_1, B_2 *lie on a circle* (O_3),
points A_2, A_3, B_2, B_3 *lie on a circle* (O_1),
points A_3, A_4, B_3, B_4 *lie on a circle* (O_2),
points A_4, A_5, B_4, B_5 *lie on a circle* (O_6),
points A_5, A_6, B_5, B_6 *lie on a circle* (O_4),
and points A_6, A_1, B_6, B_1 *lie on a circle* (O_5).
Then the lines O_1O_4, O_2O_5 *and* O_3O_6 *are concurrent.*

Figure 1 illustrates the nine-circles problem.

Special case of theorem 1. If $A_i = B_i$ for $i = 1, ..., 6$, then Theorem 1 is the special case of the Brianchon theorem. (See [3, page 77]).

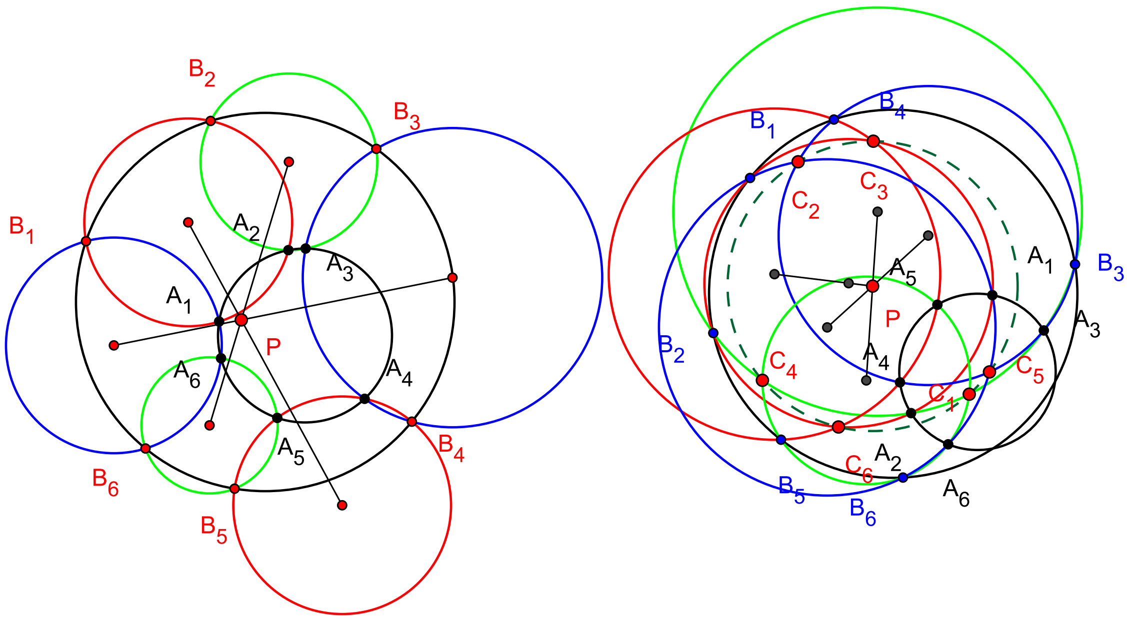

FIGURE 1. Nine Circles Problem

Problem 1. *(*[2, Nine circles problem]*) Let $C_1, C_4 = (O_1) \cap (O_4)$; $C_2, C_5 = (O_2) \cap (O_5)$; $C_3, C_6 = (O_3) \cap (O_6)$. Then points C_1, C_2, C_3, C_4, C_5 and C_6 lie on a circle whose center is the point of concurrence defined in Theorem 1.*

Special case of problem 1. If $B_1 \equiv B_2 \equiv B_3 \equiv B_4 \equiv B_5 \equiv B_6 \equiv \infty$, then problem 1 is a special case of the Pascal theorem. (See [3, page 74]).

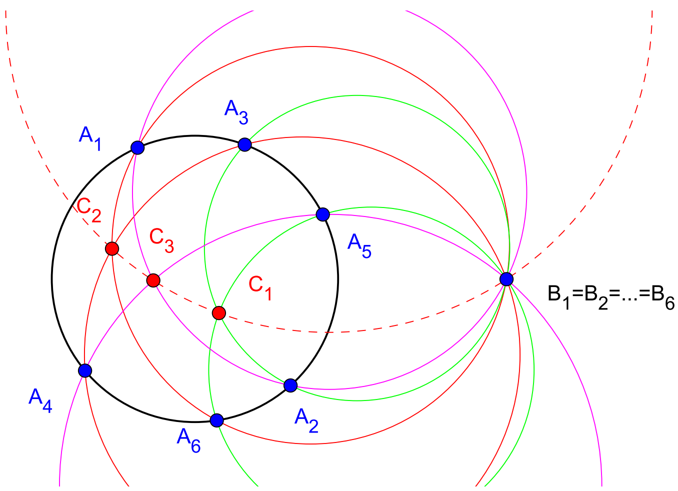

FIGURE 2. Pascal circle

2. THE SIXTEEN POINTS CIRCLE

We refer the reader about the Bundle's theorem configuration to [4, page 61] or [5, Bundle theorem].

Consider the Bundle theorem configuration :
Points A_1, A_2, A_3, A_4 lie on a circle,

points B_1, B_2, B_3, B_4 lie on a circle,
points A_1, A_2, B_1, B_2 lie on a circle,
points B_1, B_2, A_3, A_4 lie on a circle,
points B_3, B_4, A_3, A_4 lie on a circle,
and points A_3, A_4, A_1, A_2 lie on a circle.

Let the pair of circles $(P_1P_3Q_i)$ and $(P_2P_4Q_j)$ is such that if $P = A$ then $Q = B$ or if $P = B$ then $Q = A$ and if $i = 1$ then $j = 2$ or if $i = 2$ then $j = 1$ or if $i = 3$ then $j = 4$ or if $i = 4$ then $j = 3$. Hence, there are eight pair of circles with this definition.

With one pair of circles $(P_1P_3Q_i)$ and $(P_2P_4Q_j)$ we have two common points. Hence, we have 16 points of intersection of 8 pairs of circles.

Problem 2. *The eight lines passing through the centers of circles* $(P_1P_3Q_i)$ *and* $(P_2P_4Q_j)$ *are concurrent.*

Denote by O the point of concurrence of Problem 2.

Problem 3. *The sixteen points of intersection of the eight pairs of circles lie on a circle with center* O.

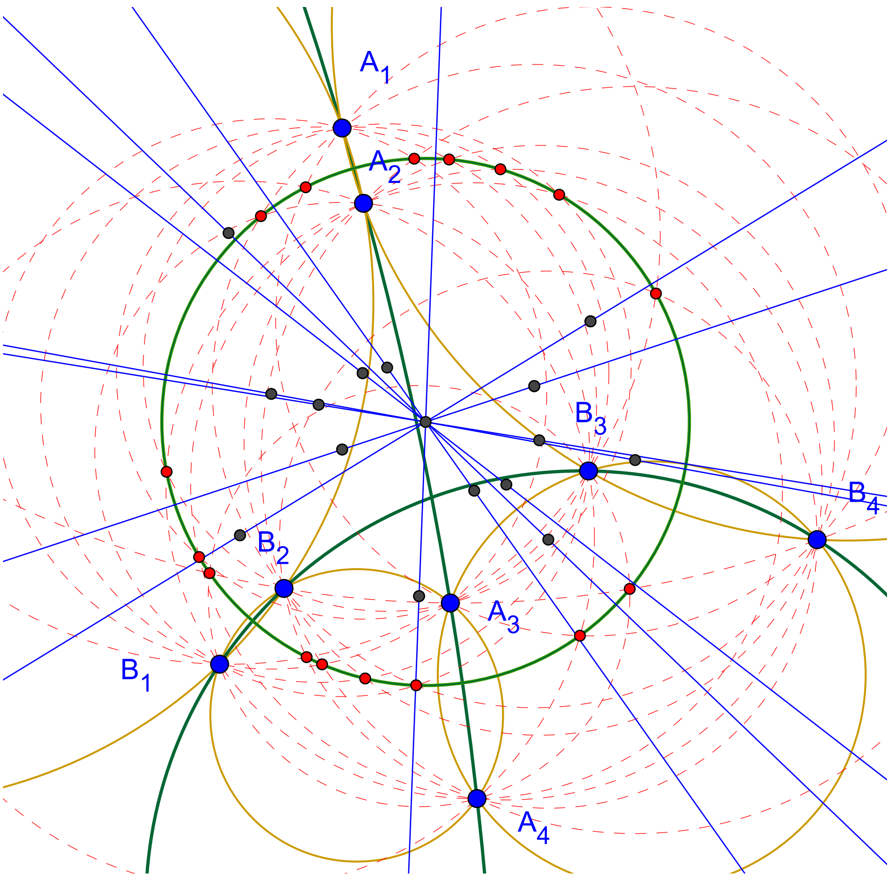

FIGURE 3. The 16 points circle associated with the Bundle theorem configuration.

Figure 3 illustrates the sixteen points circle described in Problem 3.

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