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Computer Discovered Mathematics: Lester Circles

SAVA GROZDEV^a AND DEKO DEKOV^{b2}

^a VUZF University of Finance, Business and Entrepreneurship,
Gusla Street 1, 1618 Sofia, Bulgaria

e-mail: sava.grozdev@gmail.com

^bZahari Knjazheski 81, 6000 Stara Zagora, Bulgaria

e-mail: ddekov@ddekov.eu

web: <http://www.ddekov.eu/>

Abstract. A circle is a Lester circle, if it contains at least four remarkable points of the triangle. By using the computer program “Discoverer”, we investigate Lester circles. **Keywords.** Lester circle, remarkable point, triangle geometry, computer-discovered mathematics, Euclidean geometry, “Discoverer”.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. INTRODUCTION

From the June Lester web site [8]: “There is always a circle through three given points, as long as they are not on a line. Circles through four given points - those are exceptional. Especially when the four points are well-known special points of a triangle.”

June Lester has investigated the circle on which the Circumcenter, Nine-Point Center, and the Outer and Inner Fermat points lie. Now this circle is known as the Lester circle. See [9], [11, Lester Circle], [12, Lester’s theorem].

In 2002 Bernard Gibert [2] discovered that the point Alrescha, that is, point X(1117) in the Kimberling’s ETC [7], also lies on the Lester circle.

In 2014 Grozdev and Dekov discovered, with the help of the computer program “Discoverer”, three new remarkable points which lie on the Lester circle. See [4], [5] and [6]. These new points are not available in the Kimberling’s ETC [7].

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²Corresponding author

In general, we say that a circle is a *Lester circle*, if it contains at least four remarkable points. In triangle geometry there are many Lester circles. A method for construction of Lester circles is given by Stothers [10]. By using this method Stothers has constructed hundreds Lester Circles. A list of Lester circles also is given by Nikolaos Dergiades [1].

We denote:

I is the Incenter,
 G is the Centroid,
 O is the Circumcenter,
 H is the Orthocenter,
 N is the None-Point Center,
 K is the Symmedian Point.

2. THE POINT ALRESCHA

Point Alrescha, now point X(1117) in [7], is discovered by Bernard Gibert in 2002. It is isogonal conjugate of inverse-in-circumcircle of isogonal conjugate of the Parry Reflection Point.

The “Discoverer” has discovered a new property of the Point Alrescha, starting again from the Parry Reflection Point:

Theorem 2.1. *The point Alrescha is the Euler Reflection Point of the Triangle of Reflections of the Parry Reflection Point in the Sidelines of Triangle ABC .*

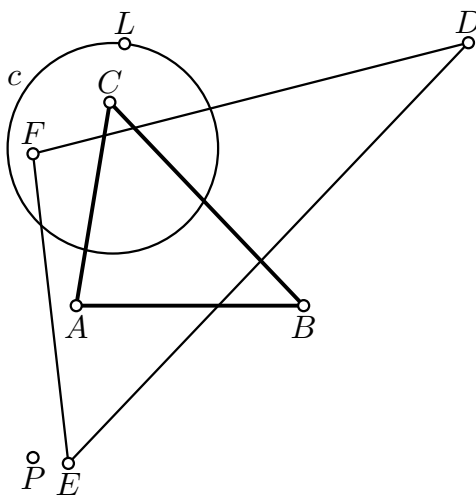


FIGURE 1.

Figure 1 illustrates the property of point Alrescha. In Figure 1, P is the Parry Reflection Point, DEF is the Triangle of Reflections of the Parry Reflection Point in the Sidelines of Triangle ABC , c is the Lester circle, E is the Euler Reflection Point of triangle DEF . Then point L is the point Alrescha. It lies on the Lester circle.

3. CIRCLES DEFINED BY THREE REMARKABLE POINTS

Paul Yiu has discovered [14, §12.2] that the circle GOK contains the Parry point X(111) and point X(691) Parry Point of the Circumcevian Triangle of the Schoute

Center. Below in theorem 3.7 we extend this result. Note that Yiu has investigated also circles OHK and ONK [14, §13]. Triplets from the points I, G, O, H, N, K define 16 circles. In this section we investigate these 16 circles, by the help of the “Discoverer” [3].

Note that we can easily find the barycentric equation of a circle defined by three points by using the formulas given by Nikolaos Dergiades. See [14, § 15].

In the theorems below, if a new point is not accompanied by its Kimberling’s number, it is not included in the [7]. The “Discoverer” has not discovered additional remarkable points on the following basic circles: IGK, IHN, IHK, INK .

Theorem 3.1. *The circle IGO contains the following additional remarkable point: Reflection of the Center of the Orthocentroidal Circle in the Inverse of the Incenter in the Orthocentroidal Circle.*

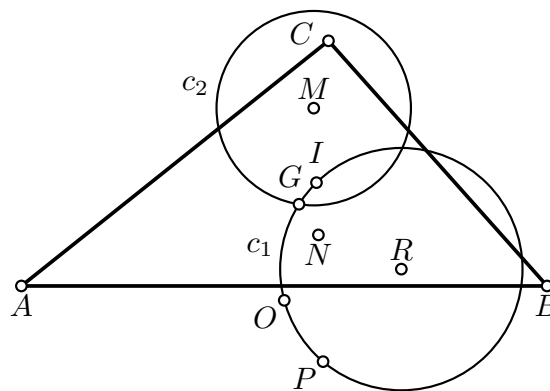


FIGURE 2.

Figure 2 illustrates Theorem 3.1. In Figure 2, I is the Incenter, G is the Centroid, O is the Circumcenter, c_1 is the circle IGO , c_2 is the Orthocentroidal circle, M is the center of c_2 , N is the inverse of I with respect to c_2 , and P is the reflection of M in N . Then point P lies on circle c_1 .

Theorem 3.2. *The circle IGH contains the following additional remarkable point: Reflection of the Inverse of the Incenter in the Orthocentroidal Circle in the Center of the Orthocentroidal Circle.*

Theorem 3.3. *The circle IGN contains the following additional remarkable point: Midpoint of the Center of the Orthocentroidal Circle and the Inverse of the Incenter in the Orthocentroidal Circle.*

Theorem 3.4. *The circle IOH contains the following additional remarkable point: Inverse of the Clawson Point in the Stevanovic Circle.*

Theorem 3.5. *The circle ION contains the following additional remarkable points:*

- (1) *Inverse of the Incenter in the Orthocentroidal Circle.*
- (2) *Parry Reflection Point of the Orthic Triangle of the Fuhrmann Triangle.*

Theorem 3.6. *The circle IOK contains the following additional remarkable points:*

- (1) *Reflection of the Parry Reflection Point in the Midpoint of the Incenter and the Symmedian Point.*

- (2) *Reflection of the Inverse of the Incenter in the Brocard Circle in the Center of the Brocard Circle.*

Theorem 3.7. *The circle GOK contains the following additional remarkable points:*

- (1) *X(111) Parry Point.*
- (2) *X(691) Parry Point of the Circumcevian Triangle of the Schoute Center.*
- (3) *Center of the Orthocentroidal Circle of the Fourth Brocard Triangle.*
- (4) *Centroid of the Circumcevian Triangle of the Schoute Center.*
- (5) *Outer Fermat Point of the Triangle of the Circumcenters of the Triangulation Triangles of the Symmedian Point.*
- (6) *Inner Fermat Point of the Triangle of the Circumcenters of the Triangulation Triangles of the Symmedian Point.*
- (7) *Center of the Orthocentroidal Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Tarry Point.*
- (8) *Reflection of the Euler Reflection Point in the Center of the Brocard Circle.*
- (9) *Quotient of the Parry Point and the Steiner Point.*
- (10) *Miquel Anticevian Point of the Parry Point.*
- (11) *Internal Center of Similitude of the Parry Circle and the Parry Circle of the Fourth Brocard Triangle.*
- (12) *External Center of Similitude of the Parry Circle and the Parry Circle of the Fourth Brocard Triangle.*

Theorem 3.8. *The circle GHK contains the following additional remarkable points:*

- (1) *X(1560) Complement of the Isotomic Conjugate of the Inverse of the Centroid in the Nine-Point Circle.*
- (2) *X(5099) Radical Center of the Nine-Point Circles of the Triangulation Triangles of the Far-Out Point.*
- (3) *Orthocenter of the Fourth Brocard Triangle of the Fourth Brocard Triangle.*
- (4) *Kiepert Center of the Honsberger Triangle of the Fourth Brocard Triangle.*
- (5) *Midpoint of the Steiner Point and the Reflection of the Centroid in the Orthocenter.*

Theorem 3.9. *The circle GNK contains the following additional remarkable points:*

- (1) *Kiepert Center of the Euler Triangle of the Center of the Orthocentroidal Circle.*
- (2) *Nine-Point Center of the Fourth Brocard Triangle of the Fourth Brocard Triangle.*
- (3) *Midpoint of the Schoute Center and the Inverse of the Centroid in the Nine-Point Circle.*
- (4) *Inverse of the Parry Point in the Nine-Point Circle.*

Theorem 3.10. *The circle OHK contains the following additional remarkable points:*

- (1) *X(112) Product of the Euler Reflection Point and the Orthocenter.*
- (2) *X(6033) Steiner Point of the Neuberg Triangle.*
- (3) *Orthocenter of the Honsberger Triangle of the Fourth Brocard Triangle.*

- (4) *Midpoint of the Parry Reflection Point and the Reflection of the Circumcenter in the Symmedian Point.*
- (5) *Reflection of the Inverse of the Orthocenter in the Brocard Circle in the Center of the Brocard Circle.*

Theorem 3.11. *The circle ONK contains the following additional remarkable points:*

- (1) *X(115) Kiepert Center.*
- (2) *Reflection of the Inverse of the Nine-Point Center in the Brocard Circle in the Center of the Brocard Circle.*

Theorem 3.12. *The circle HNK contains the following additional remarkable point: Tarry Point of the Medial Triangle of the Euler Triangle.*

4. CIRCLES DEFINED BY TWO REMARKABLE POINTS AS DIAMETER

There are two famous circles defined as endpoints of the diameters of pairs of basic circles: the Orthocentroidal circle GH and the Brocard circle OK .

In accordance with [11, Orthocentroidal Circle], the Orthocentroidal circle does not pass through any notable triangle centers other than G and H . Below we extend this result.

It is known [11, Brocard Circle] that the Brocard circles passes through Firsts and Second Brocard points, as well as through the Kimberling's centers $X(1083)$ and $X(1316)$. Below we extend this result.

There are fifteen circles formed by basic points as diameters. Below we investigate some remarkable points on these circles. In the theorems below, if a new point is not accompanied by its Kimberling's number, it is not included in the [7].

Note that we can easily find the barycentric equation of a circle PQ defined by the endpoints P and Q of the diameter. Find M , the midpoint of P and Q , and find the distance R from M to P . Then the equation is as follows: $|MX|^2 - R^2 = 0$ where $X = (x, y, z)$ are the unknowns.

Theorem 4.1. *The circle IG contains the following additional remarkable point: X(3109) Gibert Point of the Half-Circumcevian Triangle of the Incenter.*

Theorem 4.2. *The circle IO contains the following additional remarkable points:*

- (1) *X(3109) Gibert Point of the Half-Circumcevian Triangle of the Incenter.*
- (2) *X(6789) Euler Reflection Point of the Half-Circumcevian Triangle of the Spieker Center.*
- (3) *Tarry Point of the Half-Circumcevian Triangle of the Incenter.*
- (4) *Steiner Point of the Half-Circumcevian Triangle of the Incenter.*
- (5) *Parry Point of the Half-Circumcevian Triangle of the Incenter.*
- (6) *Euler Reflection Point of the Half-Circumcevian Triangle of the Second de Villiers Point.*

Theorem 4.3. *The circle IH contains the following additional remarkable points:*

- (1) *X(3109) Gibert Point of the Half-Circumcevian Triangle of the Incenter.*
- (2) *Kiepert Center of the Fuhrmann Triangle of the Antimedial Triangle.*

Theorem 4.4. *The circle IN contains the following additional remarkable point: $X(3109)$ The Gibert Point of the Half-Circumcevian Triangle of the Incenter.*

Theorem 4.5. *The circle IK contains the following additional remarkable points:*

- (1) $X(5091)$ Feuerbach Point of the First Brocard Triangle of the Antimedial Triangle.
- (2) Center of the Fuhrmann Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the First Brocard Point.
- (3) Center of the Fuhrmann Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the Second Brocard Point.
- (4) Incenter of the Orthic Triangle of the Half-Symmedian Triangle.

Theorem 4.6. *The circle GO contains the following additional remarkable points:*

- (1) $X(3111)$ Centroid of the Half-Circumcevian Triangle of the First Beltrami Point.
- (2) $X(5108)$ Parry Point of the First Brocard Triangle.
- (3) $X(6789)$ Euler Reflection Point of the Half-Circumcevian Triangle of the Spieker Center.
- (4) Euler Reflection Point of the Euler Triangle of the Tarry Point.
- (5) Parry Point of the Euler Triangle of the Steiner Point.
- (6) Tarry Point of the Half-Circumcevian Triangle of the Centroid.
- (7) Steiner Point of the Half-Circumcevian Triangle of the Centroid.
- (8) Euler Reflection Point of the Half-Circumcevian Triangle of the Outer Napoleon Point.
- (9) Euler Reflection Point of the Half-Circumcevian Triangle of the Inner Napoleon Point.
- (10) Euler Reflection Point of the Half-Circumcevian Triangle of the Third Brocard Point.
- (11) Euler Reflection Point of the Half-Circumcevian Triangle of the Outer Vecten Point.
- (12) Euler Reflection Point of the Half-Circumcevian Triangle of the Inner Vecten Point.
- (13) Centroid of the Half-Circumcevian Triangle of the First Brocard Point.
- (14) Centroid of the Half-Circumcevian Triangle of the Second Brocard Point.
- (15) $X(6789)$ Euler Reflection Point of the Half-Circumcevian Triangle of the Spieker Center.
- (16) Gibert Point of the Half-Circumcevian Triangle of the Centroid.
- (17) Euler Reflection Point of the First Brocard Triangle of the Honsberger Triangle.
- (18) Inverse of the Center of the Radical Circle of the Neuberg Circles in the Second Brocard Circle.

Theorem 4.7. *The circle GH (The Orthocentroidal circle) contains the following additional remarkable points:*

- (1) $X(6792)$ Euler Reflection Point of the Fourth Brocard Triangle.
- (2) Tarry Point of the Fourth Brocard Triangle.
- (3) Steiner Point of the Fourth Brocard Triangle.
- (4) Gibert Point of the Fourth Brocard Triangle.
- (5) Parry Point of the Triangle of the Orthocenters of the Triangulation Triangles of the Tarry Point.

- (6) *Euler Reflection Point of the Triangle of the Orthocenters of the Triangulation Triangles of the Steiner Point.*
- (7) *Incenter of the Orthic Triangle of the Fourth Brocard Triangle.*
- (8) *Feuerbach Point of the Excentral Triangle of the Fourth Brocard Triangle.*
- (9) *Feuerbach Point of the Antimedial Triangle of the Fourth Brocard Triangle.*
- (10) *Feuerbach Point of the Hexyl Triangle of the Fourth Brocard Triangle.*
- (11) *Kiepert Center of the Excentral Triangle of the Fourth Brocard Triangle.*
- (12) *Kiepert Center of the Hexyl Triangle of the Fourth Brocard Triangle.*
- (13) *Perspector of the Fourth Brocard Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Kiepert Center.*
- (14) *Inverse of the Centroid in the Stevanovic Circle.*
- (15) *Inverse of the Orthocenter in the Stevanovic Circle.*
- (16) *Inverse of the Centroid in the Lester Circle.*
- (17) *Inverse of the Orthocenter in the Lester Circle.*

Theorem 4.8. *The circle GN contains the following additional remarkable points:*

- (1) *Parry Point of the First Brocard Triangle of the Medial Triangle.*
- (2) *Tarry Point of the Fourth Brocard Triangle of the Half-Median Triangle.*
- (3) *Steiner Point of the Fourth Brocard Triangle of the Half-Median Triangle.*
- (4) *Gibert Point of the Fourth Brocard Triangle of the Half-Median Triangle.*

Theorem 4.9. *The circle GK contains the following additional remarkable points:*

- (1) *Euler Reflection Point of the First Brocard Triangle.*
- (2) *Centroid of the Half-Circumcevian Triangle of the First Beltrami Point.*
- (3) *Center of the Orthocentroidal Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the First Brocard Point.*
- (4) *Center of the Orthocentroidal Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the Second Brocard Point.*

Theorem 4.10. *The circle OH contains the following additional remarkable points:*

- (1) *Tarry Point of the Half-Circumcevian Triangle of the Orthocenter.*
- (2) *Steiner Point of the Half-Circumcevian Triangle of the Orthocenter.*
- (3) *Parry Point of the Half-Circumcevian Triangle of the Orthocenter.*
- (4) *Gibert Point of the Half-Circumcevian Triangle of the Orthocenter.*

Theorem 4.11. *The circle ON contains the following additional remarkable points:*

- (1) *Euler Reflection Point of the Euler Triangle of the Gibert Point.*
- (2) *Tarry Point of the Half-Circumcevian Triangle of the Nine-Point Center.*
- (3) *Steiner Point of the Half-Circumcevian Triangle of the Nine-Point Center.*
- (4) *Euler Reflection Point of the Half-Circumcevian Triangle of the Nine-Point Center.*
- (5) *Parry Point of the Half-Circumcevian Triangle of the Nine-Point Center.*
- (6) *Gibert Point of the Half-Circumcevian Triangle of the Nine-Point Center.*

Theorem 4.12. *The circle OK (The Brocard circle) contains the following additional remarkable points:*

- (1) *First Brocard Point.*
- (2) *Second Brocard Point.*

- (3) $X(1316)$ Euler Reflection Point of the First Brocard Triangle.
- (4) $X(5108)$ Parry Point of the First Brocard Triangle.
- (5) $X(5091)$ Feuerbach Point of the First Brocard Triangle of the Antimedial Triangle.
- (6) Gibert Point of the First Brocard Triangle.
- (7) Tarry Point of the Second Brocard Triangle.
- (8) Steiner Point of the Second Brocard Triangle.
- (9) Euler Reflection Point of the Second Brocard Triangle.
- (10) Parry Point of the Second Brocard Triangle.
- (11) Gibert Point of the Second Brocard Triangle.
- (12) Symmedian Point of the Antipedal Triangle of the First Brocard Point.
- (13) Symmedian Point of the Antipedal Triangle of the Second Brocard Point.
- (14) Incenter of the Orthic Triangle of the First Brocard Triangle.
- (15) Feuerbach Point of the Excentral Triangle of the First Brocard Triangle.
- (16) Feuerbach Point of the Hexyl Triangle of the First Brocard Triangle.
- (17) Feuerbach Point of the Excentral Triangle of the Second Brocard Triangle.
- (18) Feuerbach Point of the Antimedial Triangle of the Second Brocard Triangle.
- (19) Feuerbach Point of the Hexyl Triangle of the Second Brocard Triangle.
- (20) Kiepert Center of the Excentral Triangle of the First Brocard Triangle.
- (21) Kiepert Center of the Hexyl Triangle of the First Brocard Triangle.
- (22) Kiepert Center of the Excentral Triangle of the Second Brocard Triangle.
- (23) Kiepert Center of the Hexyl Triangle of the Second Brocard Triangle.
- (24) Anticomplement of the Isotomic Conjugate of the Complement of the Cyclocevian Conjugate of the Symmedian Point.
- (25) Inverse of the First Brocard Point in the Parry Circle.
- (26) Inverse of the Second Brocard Point in the Parry Circle.

Theorem 4.13. *The circle HN contains the following additional remarkable point: Steiner Point of the Fuhrmann Triangle of the Half-Median Triangle.*

Theorem 4.14. *The circle HK contains the following additional remarkable points:*

- (1) $X(1316)$ Euler Reflection Point of the First Brocard Triangle.
- (2) $X(6792)$ Euler Reflection Point of the Fourth Brocard Triangle.
- (3) Nine-Point Center of the Triangle of Reflections of the First Brocard Point in the Sidelines of Triangle ABC .
- (4) Nine-Point Center of the Triangle of Reflections of the Second Brocard Point in the Sidelines of Triangle ABC .
- (5) Tarry Point of the Half-Circumcevian Triangle of the Orthocenter.

Theorem 4.15. *The circle NK contains the following additional remarkable points:*

- (1) $X(1316)$ Parry Point of the First Brocard Triangle of the Medial Triangle.
- (2) Euler Reflection Point of the First Brocard Triangle.
- (3) Nine-Point Center of the Triangle of the Orthocenters of the Pedal Corner Triangles of the First Brocard Point.
- (4) Nine-Point Center of the Triangle of the Orthocenters of the Pedal Corner Triangles of the Second Brocard Point.

Below is given table of centers of circles. Note that the center of circle NK is not available in Kimberling [7].

Endpoint of diameter	Endpoint of diameter	Center of circle
X(1) Incenter	X(2) Centroid	X(551)
X(1) Incenter	X(3) Circumcenter	X(1385)
X(1) Incenter	X(4) Orthocenter	X(946)
X(1) Incenter	X(5) Nine-Point Center	X(5901)
X(1) Incenter	X(6) Symmedian Point	X(1386)
X(2) Centroid	X(3) Circumcenter	X(549)
X(2) Centroid	X(4) Orthocenter	X(381)
X(2) Centroid	X(5) Nine-Point Center	X(547)
X(2) Centroid	X(6) Symmedian Point	X(597)
X(3) Circumcenter	X(4) Orthocenter	X(5)
X(3) Circumcenter	X(5) Nine-Point Center	X(140)
X(3) Circumcenter	X(6) Symmedian Point	X(182)
X(4) Orthocenter	X(5) Nine-Point Center	X(546)
X(4) Orthocenter	X(6) Symmedian Point	X(5480)
X(5) Nine-Point Center	X(6) Symmedian Point	-

TABLE 1

5. THE F_+F_-G CIRCLE

The F_+F_-G is investigated by Paul Yiu [14].

Theorem 5.1. *The F_+F_-G circle, defined by the Centroid, Outer Fermat Point and Inner Fermat Point, contains the following additional remarkable points:*

- (1) $X(111)$ Parry Point.
- (2) $X(476)$ Tripole of the Fermat Line.
- (3) $X(5466)$ Quotient of the Parry Point and the Euler Reflection Point.
- (4) $X(5640)$ Centroid of the Pedal Triangle of the Center of the Orthocentroidal Circle.
- (5) $X(6032)$ Centroid of the Fourth Brocard Triangle.
- (6) $X(6792)$ Euler Reflection Point of the Fourth Brocard Triangle.
- (7) Centroid of the Circumcevian Triangle of the Far-Out Point.
- (8) Parry Point of the Triangle of Reflections of the Centroid in the Sidelines of Triangle ABC .
- (9) Centroid of the Triangle of Reflections of the Far-Out Point in the Sidelines of Triangle ABC .
- (10) Centroid of the Inner Lucas Triangle of the Fourth Brocard Triangle.
- (11) Feuerbach Point of the Lucas Central Triangle of the Fourth Brocard Triangle.
- (12) Far-Out Point of the Inner Lucas Triangle of the Fourth Brocard Triangle.
- (13) Euler Reflection Point of the Inner Lucas Triangle of the Fourth Brocard Triangle.
- (14) Parry Point of the Inner Lucas Triangle of the Fourth Brocard Triangle.
- (15) Weill Point of the Lucas Central Triangle of the Fourth Brocard Triangle.
- (16) Schroder Point of the Lucas Central Triangle of the Fourth Brocard Triangle.

6. THE PARRY CIRCLE

The Parry Circle is investigated by Paul Yiu [14]. See also [11, Parry Circle].

Theorem 6.1. *The Parry Circle, defined by the Centroid, First Isodynamic Point and Second Isodynamic Point, contains the following additional remarkable points:*

- (1) $X(23)$ Far-Out Point.
- (2) $X(110)$ Euler Reflection Point.
- (3) $X(111)$ Parry Point.
- (4) $X(352)$ Far-Out Point of the Circum-Symmedial Triangle.
- (5) $X(353)$ Centroid of the Circum-Symmedial Triangle.
- (6) $X(7598)$ Feuerbach Point of the Lucas Central Triangle.
- (7) $X(7601)$ Euler Reflection Point of the Inner Lucas Triangle.
- (8) Centroid of the Inner Lucas Triangle.
- (9) Far-Out Point of the Inner Lucas Triangle.
- (10) Parry Point of the Inner Lucas Triangle.
- (11) Weill Point of the Lucas Central Triangle.
- (12) Schroder Point of the Lucas Central Triangle.
- (13) Parry Point of the Antipedal Triangle of the First Isodynamic Point.
- (14) Parry Point of the Antipedal Triangle of the Second Isodynamic Point.
- (15) Parry Point of the Inner Monge Triangle of the Lucas Circles.
- (16) Centroid of the Inner Lucas Triangle of the Inner Lucas Triangle.
- (17) Feuerbach Point of the Lucas Central Triangle of the Inner Lucas Triangle.
- (18) Euler Reflection Point of the Neuberg Triangle of the Antimedial Triangle.
- (19) Far-Out Point of the Inner Lucas Triangle of the Inner Lucas Triangle.
- (20) Euler Reflection Point of the Inner Lucas Triangle of the Inner Lucas Triangle.
- (21) Parry Point of the Inner Lucas Triangle of the Inner Lucas Triangle.
- (22) Weill Point of the Lucas Central Triangle of the Inner Lucas Triangle.
- (23) Schroder Point of the Lucas Central Triangle of the Inner Lucas Triangle.
- (24) Reflection of the Centroid in the Center of the Parry Circle.
- (25) Reflection of the First Isodynamic Point in the Center of the Parry Circle.
- (26) Reflection of the Second Isodynamic Point in the Center of the Parry Circle.
- (27) Reflection of the Far-Out Point in the Center of the Parry Circle.
- (28) Reflection of the Euler Reflection Point in the Center of the Parry Circle.
- (29) Reflection of the Parry Point in the Center of the Parry Circle.
- (30) Inverse of the Far-Out Point in the Brocard Circle.
- (31) Inverse of the Centroid in the Radical Circle of the Lucas Circles.
- (32) Inverse of the Far-Out Point in the Radical Circle of the Lucas Circles.
- (33) Inverse of the Centroid in the Inner Apollonius Circle of the Lucas Circles.
- (34) Inverse of the Far-Out Point in the Inner Apollonius Circle of the Lucas Circles.

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