

International Journal of Computer Discovered Mathematics (IJCDM)  
ISSN 2367-7775 ©IJCDM  
March 2016, Volume 1, No.1, pp. 89-92  
Received 15 February 2016. Published on-line 20 February 2016  
web: <http://www.journal-1.eu/>  
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## Mathematics Discovered by Computers: Incenters of Triangles

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**Abstract.** The table in Weisstein [2, Incenter] summarizes 21 incenters for named triangles that are Kimberling centers. In this note by using the computer program “Discoverer” we extend the Weisstein’s table to 98 incenters.

**Keywords.** incenter of a triangle, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, “Discoverer”.

**Mathematics Subject Classification (2010).** 51-04, 68T01, 68T99.

### 1. INTRODUCTION

The table in Weisstein [2, Incenter] summarizes 21 incenters for named triangles that are Kimberling centers. In this note by using the computer program “Discoverer” we extend the Weisstein’s table to 98 incenters. We recommend the reader to prove the results of this paper, possibly in the form of problems.

The labeling of triangle centers follows Kimberling’s ETC [1]. For example, X(1) denotes the Incenter, X(2) denotes the Centroid, X(37) is the Grinberg Point, X(75) is the Moses point (Note that in the prototype of the “Discoverer” the Moses point is the X(35)), etc.

The reader may find the definitions in [2].

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## 2. THEOREM

**Theorem 2.1.** *The Incenter of the*

- (1) *Medial Triangle is the X(10) Spieker Center.*
- (2) *Intouch Triangle is the X(177) Incenter of the Intouch Triangle.*
- (3) *Excentral Triangle is the X(164) Incenter of the Excentral Triangle.*
- (4) *Antimedial Triangle is the X(8) Nagel Point.*
- (5) *Euler Triangle is the X(946).*
- (6) *Inner Grebe Triangle is the X(3641).*
- (7) *Outer Grebe Triangle is the X(3640).*
- (8) *Lucas Central Triangle is the X(1151) Radical Center of the Lucas Circles.*
- (9) *Malfatti Central Triangle is the X(483) Malfatti-Steiner Point.*
- (10) *Johnson Triangle is the X(355) Center of the Fuhrmann Circle.*
- (11) *Inner Yff Triangle is the X(1) Incenter.*
- (12) *Outer Yff Triangle is the X(1) Incenter.*
- (13) *First Brocard Triangle is the X(3923).*
- (14) *Half-Median Triangle is the X(1125).*
- (15) *Pedal Triangle of the First Isodynamic Point is the X(396).*
- (16) *Pedal Triangle of the Second Isodynamic Point is the X(395).*
- (17) *Antipedal Triangle of the Outer Fermat Point is the X(5463).*
- (18) *Antipedal Triangle of the Inner Fermat Point is the X(5464).*
- (19) *Circumcevian Triangle of the Circumcenter is the X(40) Bevan Point.*
- (20) *Circumcevian Triangle of the First Isodynamic Point is the X(3) Circumcenter.*
- (21) *Circumcevian Triangle of the Second Isodynamic Point is the X(3) Circumcenter.*
- (22) *Circum-Anticevian Triangle of the Centroid is the X(4859).*
- (23) *Orthic Triangle of the Incentral Triangle is the X(500) Orthocenter of the Incentral Triangle.*
- (24) *Orthic Triangle of the Medial Triangle is the X(4668).*
- (25) *Euler Triangle of the Medial Triangle is the X(6684).*
- (26) *Johnson Triangle of the Medial Triangle is the X(1385).*
- (27) *Inner Yff Triangle of the Medial Triangle is the X(10) Spieker Center.*
- (28) *Outer Yff Triangle of the Medial Triangle is the X(10) Spieker Center.*
- (29) *First Brocard Triangle of the Medial Triangle is the X(3821).*
- (30) *Half-Median Triangle of the Medial Triangle is the X(3634).*
- (31) *Orthic Triangle of the Orthic Triangle is the X(52) Orthocenter of the Orthic Triangle.*
- (32) *Tangential Triangle of the Orthic Triangle is the X(5) Nine-Point Center.*
- (33) *Kosnita Triangle of the Orthic Triangle is the X(5) Nine-Point Center.*
- (34) *Medial Triangle of the Intouch Triangle is the X(5571).*
- (35) *Orthic Triangle of the Intouch Triangle is the X(65) Orthocenter of the Intouch Triangle.*
- (36) *Intangents Triangle of the Intouch Triangle is the X(177) Incenter of the Intouch Triangle.*
- (37) *Lucas Central Triangle of the Intouch Triangle is the X(176) Inner Soddy Point.*
- (38) *Inner Yff Triangle of the Intouch Triangle is the X(177) Incenter of the Intouch Triangle.*

- (39) *Outer Yff Triangle of the Intouch Triangle is the X(177) Incenter of the Intouch Triangle.*
- (40) *Kosnita Triangle of the Intouch Triangle is the X(1) Incenter.*
- (41) *Orthic Triangle of the Extouch Triangle is the X(2321).*
- (42) *Antimedial Triangle of the Excentral Triangle is the X(167) Nagel Point of the Excentral Triangle.*
- (43) *Tangential Triangle of the Excentral Triangle is the X(40) Bevan Point.*
- (44) *Intangents Triangle of the Excentral Triangle is the X(164) Incenter of the Excentral Triangle.*
- (45) *Inner Yff Triangle of the Excentral Triangle is the X(164) Incenter of the Excentral Triangle.*
- (46) *Outer Yff Triangle of the Excentral Triangle is the X(164) Incenter of the Excentral Triangle.*
- (47) *Kosnita Triangle of the Excentral Triangle is the X(40) Bevan Point.*
- (48) *Antimedial Triangle of the Antimedial Triangle is the X(145) Nagel Point of the Anticomplementary Triangle.*
- (49) *Johnson Triangle of the Antimedial Triangle is the X(944).*
- (50) *Inner Yff Triangle of the Antimedial Triangle is the X(8) Nagel Point.*
- (51) *Outer Yff Triangle of the Antimedial Triangle is the X(8) Nagel Point.*
- (52) *Orthic Triangle of the Tangential Triangle is the X(155) Orthocenter of the Tangential Triangle.*
- (53) *Tangential Triangle of the Tangential Triangle is the X(26) Center of the Tangential Circle.*
- (54) *Kosnita Triangle of the Tangential Triangle is the X(26) Center of the Tangential Circle.*
- (55) *Inner Yff Triangle of the Euler Triangle is the X(946).*
- (56) *Outer Yff Triangle of the Euler Triangle is the X(946).*
- (57) *Orthic Triangle of the Extangents Triangle is the X(6237).*
- (58) *Orthic Triangle of the Intangents Triangle is the X(6238).*
- (59) *Inner Yff Triangle of the Inner Grebe Triangle is the X(3641).*
- (60) *Outer Yff Triangle of the Inner Grebe Triangle is the X(3641).*
- (61) *Inner Yff Triangle of the Outer Grebe Triangle is the X(3640).*
- (62) *Outer Yff Triangle of the Outer Grebe Triangle is the X(3640).*
- (63) *Tangential Triangle of the Hexyl Triangle is the X(1) Incenter.*
- (64) *Kosnita Triangle of the Hexyl Triangle is the X(1) Incenter.*
- (65) *Orthic Triangle of the Fuhrmann Triangle is the X(1) Incenter.*
- (66) *Tangential Triangle of the Fuhrmann Triangle is the X(355) Center of the Fuhrmann Circle.*
- (67) *Kosnita Triangle of the Fuhrmann Triangle is the X(355) Center of the Fuhrmann Circle.*
- (68) *Orthic Triangle of the Mixtilinear Triangle is the X(7961).*
- (69) *Tangential Triangle of the Inner Lucas Triangle is the X(6407).*
- (70) *Kosnita Triangle of the Inner Lucas Triangle is the X(6407).*
- (71) *Intangents Triangle of the Lucas Central Triangle is the X(1151) Radical Center of the Lucas Circles.*
- (72) *Inner Yff Triangle of the Lucas Central Triangle is the X(1151) Radical Center of the Lucas Circles.*
- (73) *Outer Yff Triangle of the Lucas Central Triangle is the X(1151) Radical Center of the Lucas Circles.*

- (74) *Intangents Triangle of the Malfatti Central Triangle is the X(483) Malfatti-Steiner Point.*
- (75) *Inner Yff Triangle of the Malfatti Central Triangle is the X(483) Malfatti-Steiner Point.*
- (76) *Outer Yff Triangle of the Malfatti Central Triangle is the X(483) Malfatti-Steiner Point.*
- (77) *Antimedial Triangle of the Johnson Triangle is the X(1482).*
- (78) *Inner Yff Triangle of the Johnson Triangle is the X(355) Center of the Fuhrmann Circle.*
- (79) *Outer Yff Triangle of the Johnson Triangle is the X(355) Center of the Fuhrmann Circle.*
- (80) *Euler Triangle of the Inner Yff Triangle is the X(226).*
- (81) *Johnson Triangle of the Inner Yff Triangle is the X(5252).*
- (82) *Inner Yff Triangle of the Inner Yff Triangle is the X(1) Incenter.*
- (83) *Outer Yff Triangle of the Inner Yff Triangle is the X(1) Incenter.*
- (84) *Medial Triangle of the Outer Yff Triangle is the X(1210).*
- (85) *Johnson Triangle of the Outer Yff Triangle is the X(1837).*
- (86) *Outer Yff Triangle of the Outer Yff Triangle is the X(1) Incenter.*
- (87) *Inner Yff Triangle of the First Brocard Triangle is the X(3923).*
- (88) *Outer Yff Triangle of the First Brocard Triangle is the X(3923).*
- (89) *Tangential Triangle of the Second Brocard Triangle is the X(182) Center of the Brocard Circle.*
- (90) *Kosnita Triangle of the Second Brocard Triangle is the X(182) Center of the Brocard Circle.*
- (91) *Tangential Triangle of the Outer Fermat Triangle is the X(627).*
- (92) *Kosnita Triangle of the Outer Fermat Triangle is the X(627).*
- (93) *Orthic Triangle of the Kosnita Triangle is the X(1147).*
- (94) *Tangential Triangle of the Kosnita Triangle is the X(1658).*
- (95) *Kosnita Triangle of the Kosnita Triangle is the X(1658).*
- (96) *Orthic Triangle of the Half-Bisector Triangle is the X(3945).*
- (97) *Inner Yff Triangle of the Half-Median Triangle is the X(1125).*
- (98) *Outer Yff Triangle of the Half-Median Triangle is the X(1125).*

#### REFERENCES

- [1] C. Kimberling, *Encyclopedia of Triangle Centers - ETC*, <http://faculty.evansville.edu/ck6/encyclopedia/ETC.html>.
- [2] E. W. Weisstein, *MathWorld - A Wolfram Web Resource*, <http://mathworld.wolfram.com/>