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Computer Discovered Mathematics: The Incenter

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Abstract. The Incenter is one of the classical remarkable points of the triangle. During the last 24 centuries the Incenter is a topic of investigation of many researchers. Now is the time of the computers. The computer program "Discoverer" has discovered hundreds new properties of the Incenter. In this paper we present about 200 new properties of the Incenter, most of them are new. These new properties could be used as problems for high school and university students.

Keywords. Incenter, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, "Discoverer".

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

The computer program "Discoverer", created by the authors, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science. See [?].

The Incenter is one of the classical remarkable points of the triangle. During the last 24 centuries the Incenter is a topic of investigation of many researchers. Now is the time of the computers. The computer program "Discoverer" has discovered hundreds new properties of the Incenter. In this paper we present about 200 new properties of the Incenter, most of which are new. These new properties could be used as problems for high school and university students.

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The Incenter is the point of concurrence of the interior angle bisectors of ABC. This point also serves as the center of the circle inscribed in triangle ABC (the Incircle).

In this paper by using the "Discoverer" we investigate the properties of the Incenter. We expect that the majority of the given theorems are new, discovered by a computer. Note that the "Discoverer" has discovered a number of roles the Incenter, which are not included in this paper. E.g., perspectors, similitude centers etc.

Many of the proofs of theorem are not presented here. We recommend the reader to find the proofs.

The results in this paper could extend the article "X(1) Incenter" in the Kimberling's Encyclopedia of Triangle Centers ETC [7], as well as the results in the article "Incenter" in [11].

We use barycentric coordinates. We refer the reader to [12],[2],[1],[8],[5],[6],[10]. The reader may find definitions in [4], Contents, Definitions, and in [11].

The reference triangle ABC has vertices $A = (1, 0, 0), B(0, 1, 0)$ and C(0,0,1). The side lengths of $\triangle ABC$ are denoted by $a = BC, b = CA$ and $c = AB$.
Given a point $P(u, v, w)$. Then P is *finite*, if $u + v + w \neq 0$. A finite point P is *normalized*, if $u + v + w = 1$. A finite point could be put in normalized form as follows: $P = (\frac{u}{s}, \frac{v}{s}, \frac{w}{s})$, where $s = u + v + w$.

Given point R with barycentric coordinates p, q, r with respect to triangle DEF, $D = (u_1, v_1, w_1)$, $E = (u_2, v_2, w_2)$, $F = (u_3, v_3, w_3)$. If points D, E, F and R are normalized, the barycentric coordinates u, v, w of R with respect to triangle ABC are as follows:

$$u = u_1p + u_2q + u_3r,$$
$$v = v_1p + v_2q + v_3r,$$
$$w = w_1p + w_2q + w_3r.$$

2. An Example: The Incenter as Nagel Point

Theorem 2.6.1 below states that:

The Incenter is the Nagel Point with respect to the Medial Triangle.

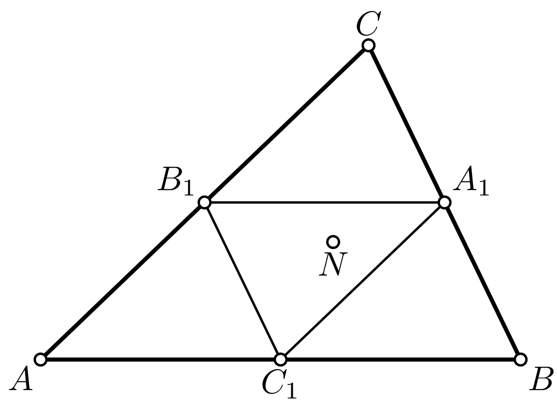

Figure 1.

Figure 1 illustrates the theorem. In Figure 1, triangle $A_1B_1C_1$ is the Medial triangle. Point N is the Incenter with respect to triangle ABC. At the same time, point N is the Nagel point with respect to triangle $A_1B_1C_1$. Hence point N has two roles: as Incenter and as Nagel Point.

Proof of the theorem.

Denote by a, b, c the sidelengths of triangle ABC. The barycentric coordinates of the Medial triangle $A_1B_1C_1$ are as follows: $A_1 = (0, 1, 1)$, $B_1 = (1, 0, 1)$, $C_1 = (1, 1, 0)$. The sidelengths of triangle $A_1B_1C_1$ are as follows: $\frac{a}{2}, \frac{b}{2}, \frac{c}{2}$. Point N has barycentric coordinates:

$$N = (b + c - a, c + a - b, a + b - c).$$

Now we use the formula for transformation of the barycentric coordinates of a point, given in the Introduction, and we obtain that point N with respect to triangle ABC has barycentric coordinates (a, b, c). Hence, with respect to triangle ABC the point is the Incenter and with respect to triangle $A_1B_1C_1$ the point is the Nagel point.

3. ROLES WITH RESPECT TO TRIANGLES

Theorem 3.1. *The Incenter is the Incenter with respect to the*

(1) *Inner Yff Triangle.*
(2) *Outer Yff Triangle.*
(3) *Triangle of the Orthocenters of the Triangulation Triangles of the Orthocenter.*
(4) *Triangle of the Incenters of the Cevian Corner Triangles of the Centroid.*
(5) *Triangle of the Nagel Points of the Anticevian Corner Triangles of the Centroid.*
(6) *Triangle of the Circumcenters of the Pedal Corner Triangles of the Incenter.*
(7) *Triangle of the Nagel Points of the Antipedal Corner Triangles of the Orthocenter.*
(8) *Kosnita Triangle of the Intouch Triangle.*
(9) *Tangential Triangle of the Hexyl Triangle.*
(10) *Inner Napoleon Point of the Outer Fermat Triangle of the Hexyl Triangle.*
(11) *Kosnita Triangle of the Hexyl Triangle.*
(12) *Orthic Triangle of the Fuhrmann Triangle.*
(13) *Inner Yff Triangle of the Inner Yff Triangle.*
(14) *Outer Yff Triangle of the Inner Yff Triangle.*
(15) *Outer Yff Triangle of the Outer Yff Triangle.*

Theorem 3.2. *The Incenter is the Centroid with respect to the*

(1) *Honsberger Triangle of the Intouch Triangle.*
(2) *Honsberger Triangle of the Hexyl Triangle.*

Theorem 3.3. *The Incenter is the Circumcenter with respect to the*

(1) *Intouch Triangle.*
(2) *Hexyl Triangle.*
(3) *Triangle of Reflections of the Incenter in the Sidelines of Triangle ABC.*
(4) *Triangle of the Incenters of the Cevian Corner Triangles of the Gergonne Point.*
(5) *Triangle of the Incenters of the Pedal Corner Triangles of the Incenter.*

(6) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Center of the Fuhrmann Circle.*
(7) *Antimedial Triangle of the Excentral Triangle.*
(8) *Antimedial Triangle of the Fuhrmann Triangle.*
(9) *Johnson Triangle of the Fuhrmann Triangle.*
(10) *Excentral Triangle of the Yff Central Triangle.*
(11) *Intouch Triangle of the Inner Yff Triangle.*
(12) *Hexyl Triangle of the Inner Yff Triangle.*
(13) *Intouch Triangle of the Outer Yff Triangle.*
(14) *Hexyl Triangle of the Outer Yff Triangle.*

3.1. **The Orthocenter.**

Theorem 3.4. *The Incenter is the Orthocenter with respect to the*

(1) *Excentral Triangle.*
(2) *Fuhrmann Triangle.*
(3) *Anticevian Triangle of the Center of the Stevanovic Circle.*
(4) *Circum-Incentral Triangle.*
(5) *Circumcevian Triangle of the Inverse of the Incenter in the Circumcircle.*
(6) *Triangle of Reflections of the Inverse of the Incenter in the Circumcircle in the Sidelines of Triangle ABC.*
(7) *Anticevian Euler Triangle of the Incenter.*
(8) *Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
(9) *Triangle of the Nine-Point Centers of the Triangulation Triangles of the Incenter.*
(10) *Triangle of the Incenters of the Cevian Corner Triangles of the Orthocenter.*
(11) *Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
(12) *Triangle of the Orthocenters of the Pedal Corner Triangles of the Incenter.*
(13) *Triangle of the Circumcenters of the Antipedal Corner Triangles of the Incenter.*
(14) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Bevan Point.*
(15) *Medial Triangle of the Intouch Triangle.*
(16) *Johnson Triangle of the Intouch Triangle.*
(17) *Euler Triangle of the Excentral Triangle.*
(18) *Euler Triangle of the Fuhrmann Triangle.*
(19) *Hexyl Triangle of the Yff Central Triangle.*
(20) *Excentral Triangle of the Inner Yff Triangle.*
(21) *Fuhrmann Triangle of the Inner Yff Triangle.*
(22) *Excentral Triangle of the Outer Yff Triangle.*
(23) *Fuhrmann Triangle of the Outer Yff Triangle.*

Theorem 3.5. *The Incenter is the Nine-Point Center with respect to the*

(1) *Excentral Triangle of the Intouch Triangle.*
(2) *Antimedial Triangle of the Intouch Triangle.*
(3) *Hexyl Triangle of the Intouch Triangle.*
(4) *Excentral Triangle of the Hexyl Triangle.*

(5) *Antimedial Triangle of the Hexyl Triangle.*
(6) *Hexyl Triangle of the Hexyl Triangle.*

Theorem 3.6. *The Incenter is the Nagel Point with respect to the*

(1) *Medial Triangle.*
(2) *Inner Monge Triangle of the Inner Yff Circles.*
(3) *Inner Monge Triangle of the Outer Yff Circles.*
(4) *Triangle of the Centroids of the Triangulation Triangles of the Incenter.*
(5) *Half-Anticevian Triangle of the Centroid.*
(6) *Triangle of the Orthocenters of the Anticevian Corner Triangles of the Incenter.*
(7) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Incenter.*
(8) *Half-Median Triangle of the Antimedial Triangle.*
(9) *Medial Triangle of the Inner Yff Triangle.*
(10) *Medial Triangle of the Outer Yff Triangle.*

Theorem 3.7. *The Incenter is the Spieker Center with respect to the*

(1) *Antimedial Triangle.*
(2) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Spieker Center.*
(3) *Antimedial Triangle of the Inner Yff Triangle.*
(4) *Antimedial Triangle of the Outer Yff Triangle.*

Theorem 3.8. *The Incenter is the Outer Napoleon Point with respect to the*

(1) *Inner Fermat Triangle of the Hexyl Triangle.*
(2) *Inner Fermat Triangle of the Intouch Triangle.*

Theorem 3.9. *The Incenter is the Inner Napoleon Point with respect to the*

(1) *Outer Fermat Triangle of the Intouch Triangle.*
(2) *Outer Fermat Triangle of the Hexyl Triangle.*

Theorem 3.10. *The Incenter is the de Longchamps Point with respect to the*

(1) *Circum-Anticevian Triangle of the Incenter.*
(2) *Euler Triangle of the Intouch Triangle.*
(3) *Half-Median Triangle of the Intouch Triangle.*
(4) *Medial Triangle of the Excentral Triangle.*
(5) *Half-Median Triangle of the Hexyl Triangle.*
(6) *Medial Triangle of the Fuhrmann Triangle.*
(7) *Triangle of the Orthocenters of the Pedal Corner Triangles of the Inverse of the Incenter in the Circumcircle.*

Theorem 3.11. *The Incenter is the Bevan Point with respect to the*

(1) *Yff Central Triangle.*
(2) *Circumcevian Triangle of the Circumcenter.*
(3) *Triangle of the Orthocenters of the Anticevian Corner Triangles of the Centroid.*
(4) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Orthocenter.*
(5) *Euler Triangle of the Antimedial Triangle.*

(6) *Yff Central Triangle of the Inner Yff Triangle.*
(7) *Yff Central Triangle of the Outer Yff Triangle.*
(8) *Orthic Triangle of the Intouch Triangle.*

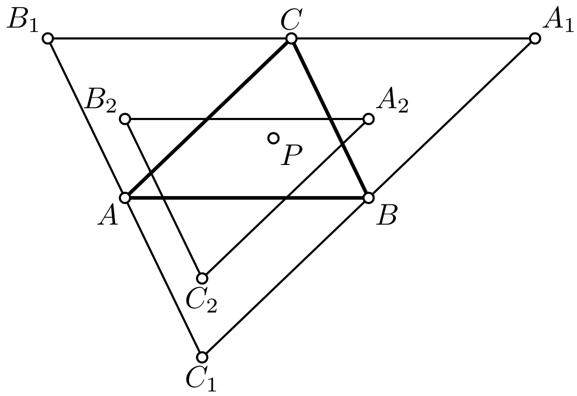

Figure 2.

Figure 2 illustrates Theorem 2.11.5. In Figure 2, $A_1B_1C_1$ is the Antimedial Triangle of triangle ABC and $A_2B_2C_2$ is the Euler triangle of triangle $A_1B_1C_1$. Point P is the Bevan Point of $A_2B_2C_2$ and at the same time P is the Incenter of triangle ABC.

Theorem 3.12. *The Incenter is the Prasolov Point with respect to the*

(1) *Triangle of the Incenters of the Anticevian Corner Triangles of the Incenter.*
(2) *Triangle of the Incenters of the Antipedal Corner Triangles of the Incenter.*

Theorem 3.13. *The Incenter is the Tarry Point with respect to the*

(1) *First Brocard Triangle of the Intouch Triangle.*
(2) *First Brocard Triangle of the Hexyl Triangle.*

Theorem 3.14. *The Incenter is the Euler Reflection Point with respect to the*

(1) *Fuhrmann Triangle of the Excentral Triangle.*
(2) *Fuhrmann Triangle of the Fuhrmann Triangle.*

Theorem 3.15. *The Incenter is the Outer Soddy Point with respect to the*

(1) *Triangle of the Incenters of the Triangulation Triangles of the Inner Soddy Point.*
(2) *Lucas Central Triangle of the Intouch Triangle.*
(3) *Lucas Central Triangle of the Hexyl Triangle.*

Figure 3 illustrates Theorem 3.16.1.In Figure 3 triangle DEF is the de Villiers triangle. The point inside triangle DEF is the Inner Vecten Point of de Villiers triangle. At the same time, this point is the Incenter of traingle ABC.

Theorem 3.16. *The Incenter is the Inner Vecten Point with respect to the*

(1) *de Villiers Triangle.*
(2) *Triangle of the Incenters of the Pedal Corner Triangles of the Inner Soddy Point.*
(3) *de Villiers Triangle of the Inner Yff Triangle.*

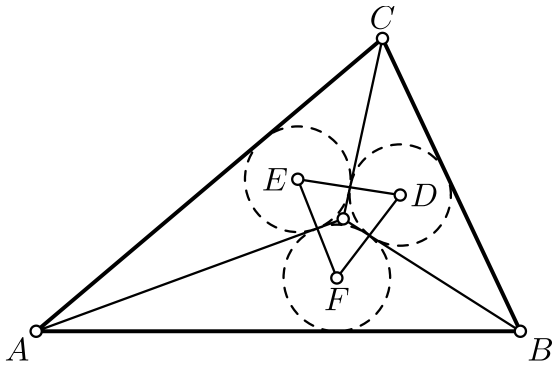

FIGURE 3.

(4) *de Villiers Triangle of the Outer Yff Triangle.*

Theorem 3.17. *The Incenter is the Center of the Tangential Circle with respect to the*

(1) *Intouch Triangle of the Intouch Triangle.*
(2) *Intouch Triangle of the Hexyl Triangle.*

Theorem 3.18. *The Incenter is the Center of the Fuhrmann Circle with respect to the*

(1) *Johnson Triangle.*
(2) *Johnson Triangle of the Inner Yff Triangle.*
(3) *Johnson Triangle of the Outer Yff Triangle.*

Theorem 3.19. *The Incenter is the Center of the Taylor Circle of the Excentral Triangle of the Antimedial Triangle.*

4. PRODUCTS AND QUOTIENTS

Theorem 4.1. *The Incenter is the Prasolov Product of the*

(1) *Centroid and the Spieker Center of the Medial Triangle.*
(2) *Gergonne Point and the Nine-Point Center of the Intouch Triangle.*
(3) *Isotomic Conjugate of the Incenter and the Spieker Center.*

Theorem 4.2. *The Incenter is the Kosnita Product of the*

(1) *Incenter and the Circumcenter.*
(2) *Reflection of the Nagel Point in the Incenter and the Centroid.*

Theorem 4.3. *The Incenter is the Stevanovic Product of the*

(1) *Incenter and the Circumcenter.*
(2) *First Isodynamic Point and the Incenter.*
(3) *Second Isodynamic Point and the Incenter.*

Theorem 4.4. *The Incenter is the Product of the*

(1) *Gergonne Point and the Mittenpunkt.*
(2) *Clawson Point and the Retrocenter.*
(3) *Second Power Point and the Third Brocard Point.*
(4) *First Jerabek Point and the Second Jerabek Point.*
(5) *Symmedian Point and the Isotomic Conjugate of the Incenter.*

(6) *Nagel Point and the Isogonal Conjugate of the Mittenpunkt.*
(7) *Schiffler Point and the Perspector of the Half-Bisector Triangle and the Intouch Triangle.*
(8) *Grinberg Point and the Isotomic Conjugate of the Spieker Center.*
(9) *Internal Center of Similitude of the Incircle and the Circumcircle and the Isotomic Conjugate of the Mittenpunkt.*

Theorem 4.5. *The Incenter is the Quotient of the*

(1) *Symmedian Point and the Incenter.*
(2) *Mittenpunkt and the Nagel Point.*
(3) *Clawson Point and the Orthocenter.*
(4) *Second Power Point and the Symmedian Point.*
(5) *Third Power Point and the Second Power Point.*
(6) *Grinberg Point and the Spieker Center.*
(7) *Internal Center of Similitude of the Incircle and the Circumcircle and the Mittenpunkt.*
(8) *Fourth Power Point and the Third Power Point.*
(9) *Gergonne Point and the Isotomic Conjugate of the Mittenpunkt.*
(10) *External Center of Similitude of the Incircle and the Circumcircle and the Isogonal Conjugate of the Mittenpunkt.*
(11) *Weill Point and the Mittenpunkt of the Medial Triangle.*
(12) *Isotomic Conjugate of the Incenter and the Third Brocard Point.*
(13) *Isogonal Conjugate of the Mittenpunkt and the Gergonne Point.*
(14) *Perspector and Homothetic Center of the Orthic Triangle and the Tangential Triangle and the Clawson Point.*
(15) *Perspector of the Excentral Triangle and the Symmedial Triangle and the Equal Parallelians Point.*

Theorem 4.6. *The Incenter is the Square Quotient of the*

(1) *Symmedian Point and the Second Power Point.*
(2) *Second Power Point and the Fourth Power Point.*
(3) *Yff Center of Conguence and the Gergonne Point.*
(4) *Malfatti-Steiner Point and the First Ajima-Malfatti Point.*
(5) *Centroid and the Isotomic Conjugate of the Incenter.*
(6) *Mittenpunkt and the Perspector of the Extouch Triangle and the Intangents Triangle.*

5. Circles containing the Incenter

Theorem 5.1. *The Incenter lies on the following circles:*

(1) *Brocard Circle of the Intouch Triangle.*
(2) *Lester Circle of the Intouch Triangle.*
(3) *Orthocentroidal Circle of the Excentral Triangle.*
(4) *Excentral Circle of the Intangents Triangle.*
(5) *Brocard Circle of the Hexyl Triangle.*
(6) *Lester Circle of the Hexyl Triangle.*
(7) *Orthocentroidal Circle of the Fuhrmann Triangle.*
(8) *Orthocentroidal Circle of the Anticevian Triangle of the Center of the Stevanovic Circle.*

(9) *Orthocentroidal Circle of the Circum-Incentral Triangle.*
(10) *Incircle of the Euler Triangle of the Feuerbach Point.*
(11) *Orthocentroidal Circle of the Triangle of the Orthocenters of the Antipedal Corner Triangles of the Bevan Point.*
(12) *Brocard Circle of the Triangle of the Orthocenters of the Antipedal Corner Triangles of the Center of the Fuhrmann Circle.*
(13) *Lester Circle of the Triangle of the Orthocenters of the Antipedal Corner Triangles of the Center of the Fuhrmann Circle.*
(14) *Circumcircle of the Fuhrmann Triangle of the Medial Triangle.*
(15) *Orthocentroidal Circle of the Medial Triangle of the Intouch Triangle.*
(16) *Lester Circle of the Excentral Triangle of the Intouch Triangle.*
(17) *Lester Circle of the Antimedial Triangle of the Intouch Triangle.*
(18) *Lester Circle of the Hexyl Triangle of the Intouch Triangle.*
(19) *Orthocentroidal Circle of the Johnson Triangle of the Intouch Triangle.*
(20) *Orthocentroidal Circle of the Honsberger Triangle of the Intouch Triangle.*
(21) *Parry Circle of the Honsberger Triangle of the Intouch Triangle.*
(22) *Brocard Circle of the Antimedial Triangle of the Excentral Triangle.*
(23) *Lester Circle of the Antimedial Triangle of the Excentral Triangle.*
(24) *Circumcircle of the Fuhrmann Triangle of the Excentral Triangle.*
(25) *Parry Circle of the Fuhrmann Triangle of the Excentral Triangle.*
(26) *Nine-Point Circle of the Fuhrmann Triangle of the Antimedial Triangle.*
(27) *Symmedial Circle of the Fuhrmann Triangle of the Antimedial Triangle.*
(28) *Lester Circle of the Excentral Triangle of the Hexyl Triangle.*
(29) *Lester Circle of the Antimedial Triangle of the Hexyl Triangle.*
(30) *Lester Circle of the Hexyl Triangle of the Hexyl Triangle.*
(31) *Orthocentroidal Circle of the Honsberger Triangle of the Hexyl Triangle.*
(32) *Parry Circle of the Honsberger Triangle of the Hexyl Triangle.*
(33) *Brocard Circle of the Antimedial Triangle of the Fuhrmann Triangle.*
(34) *Lester Circle of the Antimedial Triangle of the Fuhrmann Triangle.*
(35) *Orthocentroidal Circle of the Euler Triangle of the Fuhrmann Triangle.*
(36) *Circumcircle of the Fuhrmann Triangle of the Fuhrmann Triangle.*
(37) *Parry Circle of the Fuhrmann Triangle of the Fuhrmann Triangle.*
(38) *Brocard Circle of the Johnson Triangle of the Fuhrmann Triangle.*
(39) *Lester Circle of the Johnson Triangle of the Fuhrmann Triangle.*
(40) *Brocard Circle of the Excentral Triangle of the Yff Central Triangle.*
(41) *Lester Circle of the Excentral Triangle of the Yff Central Triangle.*
(42) *Orthocentroidal Circle of the Hexyl Triangle of the Yff Central Triangle.*
(43) *Antimedial Circle of the Fuhrmann Triangle of the Yff Central Triangle.*
(44) *Brocard Circle of the Intouch Triangle of the Inner Yff Triangle.*
(45) *Lester Circle of the Intouch Triangle of the Inner Yff Triangle.*
(46) *Orthocentroidal Circle of the Excentral Triangle of the Inner Yff Triangle.*
(47) *Excentral Circle of the Intangents Triangle of the Inner Yff Triangle.*
(48) *Brocard Circle of the Hexyl Triangle of the Inner Yff Triangle.*
(49) *Lester Circle of the Hexyl Triangle of the Inner Yff Triangle.*
(50) *Orthocentroidal Circle of the Fuhrmann Triangle of the Inner Yff Triangle.*
(51) *Brocard Circle of the Intouch Triangle of the Outer Yff Triangle.*
(52) *Lester Circle of the Intouch Triangle of the Outer Yff Triangle.*
(53) *Orthocentroidal Circle of the Excentral Triangle of the Outer Yff Triangle.*
(54) *Excentral Circle of the Intangents Triangle of the Outer Yff Triangle.*

(55) *Brocard Circle of the Hexyl Triangle of the Outer Yff Triangle.*
(56) *Lester Circle of the Hexyl Triangle of the Outer Yff Triangle.*
(57) *Orthocentroidal Circle of the Fuhrmann Triangle of the Outer Yff Triangle.*
(58) *Circle having center at the Nine-Point Center and passing through the Center of the Fuhrmann Circle.*
(59) *Circle having center at the Spieker Center and passing through the Nagel Point.*
(60) *Radical Circle of the Triad of the Antimedial Circle, the Brocard Circle and the Excentral Circle.*

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REFERENCES

[1] P. Douillet, *Translation of the Kimberling's Glossary into barycentrics*, 2012, http://www.ddekov.eu/e2/htm/links/Douillet.pdf
[2] Francisco Javier García Capitán, *Barycentric Coordinates*, International Journal of Computer Discovered Mathematics, 2015, vol. 0, no 0, 32-48. http://www.journal-1.eu/2015/01/Francisco-Javier-Barycentric-Coordinates-pp.32-48.pdf
[3] S. Grozdev and D. Dekov, *A Survey of Mathematics Discovered by Computers*, International Journal of Computer Discovered Mathematics, 2015, vol.0, no.0, 3-20. http://www.journal-1.eu/2015/01/Grozdev-Dekov-A-Survey-pp.3-20.pdf.
[4] S. Grozdev and D. Dekov, Computer-Generated Encyclopeda of Euclidean Geometry, http://www.ddekov.eu/e2/index.htm
[5] S. Grozdev and V. Nenkov, *Three Remarkable Points on the Medians of a Triangle* (Bulgarian), Sofia, Archimedes, 2012.
[6] S. Grozdev and V. Nenkov, *On the Orthocenter in the Plane and in the Space* (Bulgarian), Sofia, Archimedes, 2012.
[7] C. Kimberling, *Encyclopedia of Triangle Centers - ETC*, http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.
[8] G. Paskalev and I. Tchobanov, *Remarkable Points in the Triangle* (in Bulgarian), Sofa, Narodna Prosveta, 1985.
[9] G. Paskalev, *With coordinates in Geometry* (in Bulgarian), Sofia, Modul-96, 2000.
[10] M. Schindler and K.Cheny, Barycentric Coordinates in Olympiad Geometry, 2012, http://www.mit.edu/~evanchen/handouts/bary/bary-full.pdf
[11] E. W. Weisstein, *MathWorld - A Wolfram Web Resource*, http://mathworld.wolfram.com/
[12] P. Yiu, *Introduction to the Geometry of the Triangle*, 2001, new version of 2013, http://math.fau.edu/Yiu/YIUIntroductionToTriangleGeometry130411.pdf