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Computer Discovered Mathematics: Circles through the Feuerbach Point

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Abstract. We know that the Feuerbach point lies on the Incircle and on the Nine-Point Circle. But there are many other remarkable circle which contain the Feuerbach Point. In this note, by using the computer program “Discoverer” we find many new remarkable circle that contain the Feuerbach point. The new theorem could be used as problems for high school and university students. We recommend the reader to find the proofs.

Keywords. Feuerbach Point, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, “Discoverer”.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. INTRODUCTION

The computer program “Discoverer”, created by the authors, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science. See [?].

The Feuerbach Point is one of the famous remarkable points of the triangle. We know that the Feuerbach point lies on the Incircle and on the Nine-Point Circle. But there are many other remarkable circle which contain the Feuerbach Point. Now is the time of the computers. In this note, by using the computer program “Discoverer” we find many new remarkable circle that contain the Feuerbach point. The new theorem could be used as problems for high school and university students. We recommend the reader to find the proofs.

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The Feuerbach point is the point X(11) in the Kimberling's Encyclopedia of Triangle Centers (ETC) [1]. The results in this paper extend the article "X(11) Feuerbch Point" in [1], as well as the results in the article "Feuerbach Point" in [2].

2. THEOREMS

Theorem 2.1. *The Feuerbach Point lies on the*

- (1) *Incircle.*
- (2) *Nine-Point Circle.*
- (3) *Incentral Circle.*
- (4) *Extouch Circle.*

Theorem 2.2. *The Feuerbach Point lies on the Circumcircle of the Cevian Triangle of the*

- (1) *Mittenpunkt.*
- (2) *Schiffler Point.*
- (3) *Yff Center of Conguence.*
- (4) *First Mid-Arc Point.*

Theorem 2.3. *The Feuerbach Point lies on the Circumcircle of the Pedal Triangle of the*

- (1) *Gergonne Point.*
- (2) *Nagel Point.*
- (3) *Mittenpunkt.*
- (4) *Schiffler Point.*
- (5) *Inverse of the Incenter in the Circumcircle.*
- (6) *First Mid-Arc Point.*
- (7) *Weill Point.*
- (8) *Evans Perspector.*
- (9) *Schroder Point.*
- (10) *Bevan-Schroder Point.*
- (11) *Pohoata Point.*

Theorem 2.4. *The Feuerbach Point lies on the*

- (1) *Circle passing through the Grinberg Point, Internal Center of Similitude of the Incircle and the Circumcircle and Symmedian Point.*
- (2) *Circle passing through the Inner Soddy Point, Outer Soddy Point and Weill Point.*
- (3) *Circle passing through the Inner Oldknow Point, Outer Oldknow Point and Weill Point.*
- (4) *Circle passing through the First Rigby Point, Second Rigby Point and Weill Point.*
- (5) *Circle passing through the First Griffiths Point, Second Griffiths Point and Weill Point.*

Theorem 2.5. *The Feuerbach Point lies on the*

- (1) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter.*

- (2) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Gergonne Point.*
- (3) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Nagel Point.*
- (4) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Mittenpunkt.*
- (5) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Schiffler Point.*
- (6) *Nine-Point Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the First Mid-Arc Point.*

Theorem 2.6. *The Feuerbach Point lies on the*

- (1) *Brocard Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the Inverse of the Incenter in the Circumcircle.*
- (2) *Lester Circle of the Triangle of the Orthocenters of the Pedal Corner Triangles of the Inverse of the Incenter in the Circumcircle.*

Theorem 2.7. *The Feuerbach Point lies on the*

- (1) *Nine-Point Circle of the Triangle of the Nagel Points of the Antipedal Corner Triangles of the Tarry Point.*

Theorem 2.8. *The Feuerbach Point lies on the*

- (1) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the Bevan Point.*
- (2) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the Internal Center of Similitude of the Incircle and the Circumcircle.*
- (3) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the External Center of Similitude of the Incircle and the Circumcircle.*
- (4) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the Weill Point.*
- (5) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the Evans Perspector.*
- (6) *Circumcircle of the Stevanovic Triangle of the Incenters of the Triangulation Triangles of the Pohoata Point.*

Theorem 2.9. *The Feuerbach Point lies on the*

- (1) *Parry Circle of the Intouch Triangle.*
- (2) *Nine-Point Circle of the Outer Yff Triangle.*

Theorem 2.10. *The Feuerbach Point lies on the*

- (1) *Parry Circle of the Inner Apollonius Triangle of the Soddy Circles.*
- (2) *Symmedial Circle of the Excentral Triangle of the Medial Triangle.*
- (3) *Symmedial Circle of the Antimedial Triangle of the Intouch Triangle.*
- (4) *Parry Circle of the First Brocard Triangle of the Intouch Triangle.*
- (5) *Incircle of the Johnson Triangle of the Outer Yff Triangle.*
- (6) *Incentral Circle of the Johnson Triangle of the Outer Yff Triangle.*
- (7) *Extouch Circle of the Johnson Triangle of the Outer Yff Triangle.*

Theorem 2.11. *The Feuerbach Point lies on the Radical Circle of the Triad of the*

- (1) *Circumcircle, the Incircle and the Nine-Point Circle.*
- (2) *Incircle, the Nine-Point Circle and the Symmedial Circle.*
- (3) *Incircle, the Nine-Point Circle and the Spieker Circle.*
- (4) *Incircle, the Nine-Point Circle and the Stevanovic Circle.*
- (5) *Incircle, the Nine-Point Circle and the Sine-Triple-Angle Circle.*
- (6) *Incircle, the Nine-Point Circle and the Second Brocard Circle.*
- (7) *Incircle, the Nine-Point Circle and the Parry Circle.*
- (8) *Incircle, the Nine-Point Circle and the Outer Johnson-Yff Circle.*
- (9) *Incircle, the Nine-Point Circle and the Taylor Circle.*
- (10) *Incircle, the Moses Circle and the Nine-Point Circle.*
- (11) *Incircle, the Lester Circle and the Nine-Point Circle.*
- (12) *Incircle, the Inner Johnson-Yff Circle and the Nine-Point Circle.*
- (13) *Incircle, the Lemoine Circle and the Nine-Point Circle.*
- (14) *Incircle, the Kenmotu Circle and the Nine-Point Circle.*
- (15) *Excentral Circle, the Incircle and the Nine-Point Circle.*
- (16) *Antimedial Circle, the Incircle and the Nine-Point Circle.*
- (17) *Brocard Circle, the Incircle and the Nine-Point Circle.*
- (18) *Half-Moses Circle, the Incircle and the Nine-Point Circle.*
- (19) *Gallatly Circle, the Incircle and the Nine-Point Circle.*
- (20) *Cosine Circle, the Incircle and the Nine-Point Circle.*
- (21) *Apollonius Circle, the Incircle and the Nine-Point Circle.*

Theorem 2.12. *The Feuerbach Point lies on the*

- (1) *Circle having center at the Center of the Stevanovic Circle and passing through the Schroder Point.*

Theorem 2.13. *The Feuerbach Point lies on the*

- (1) *Parry Circle of the Pedal Triangle of the Inverse of the Incenter in the Circumcircle.*

Theorem 2.14. *The Feuerbach Point lies on the*

- (1) *Lester Circle of the Triangle of Reflections of the Inverse of the Incenter in the Circumcircle in the Sidelines of Triangle ABC.*

REFERENCES

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