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## The computer program “Inverse Matrices”

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**Abstract.** We present the computer program “Inverse Matrices”. The program gives step-by-step solution of the problem of finding of the inverse matrix of a square matrix of order 2,3,4 or 5, whose elements are integers or common fractions. The program uses the Gauss-Jordan elimination method The program is designed for university students and professors.

**Keywords.** inverses matrices, Gauss-Jordan elimination method, computer program “Inverse Matrices”.

**Mathematics Subject Classification (2010):** 15-04, 15-01.

### 1. INTRODUCTION

The computer program “Inverse Matrices” is created in 2003 by Deko Dekov. The program is designed for Windows. During the years the program has received a number of awards.

The study of inverse matrices is a part of the university education in many areas - mathematics, computer science, engineering, economics and so on. The students have to find the inverse matrix of a given matrix by using the Gauss-Jordan elimination method. The program solves this problem for square matrices of order 2,3,4 or 5, whose elements are integers or common fractions. The program provides detailed, step-by-step solution in a tutorial-like format.

In 2011 the computer program is translated in Russian by the author.

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## 2. GAUSS-JORDAN ELIMINATION METHOD

Recall a few definitions. See e.g. [2],[4],[5]. A square matrix  $A$  of order  $n$  is said to be *invertible* if there exists a matrix  $A^{-1}$  such that  $A^{-1}A = I$  and  $AA^{-1} = I$  where  $I$  is the  $n \times n$  identity matrix. If it exists,  $A^{-1}$  is called the *inverse* of  $A$ .

Let  $M$  be a  $m \times n$  matrix ( $m$  rows,  $n$  columns). There are three elementary row operations on  $M$ :

1. Type 1: Exchange two rows of  $M$ .
2. Type 2: Multiply a row of  $M$  by a nonzero constant.
3. Type 3: Add a multiple of one row of  $M$  to another row.

The Gauss-Jordan elimination method is as follows.

To determine whether a given square matrix  $A$  is invertible and to find its inverse  $A^{-1}$  if it exists, first we form the block matrix  $[A|I]$  where  $I$  is the identity matrix. Then we use the elementary row operations to perform the reduction  $[E|B]$  where  $E$  is the reduced row echelon form of  $A$ . Then:

1. If  $E = I$  the matrix  $A$  is invertible and  $A^{-1} = B$ .
2. If  $E \neq I$ ,  $A$  is not invertible, that is,  $A^{-1}$  does not exist.

## 3. AN EXAMPLE

We illustrate the use of the computer program by means of an example. We consider the case of a square 3-matrix. We enter the elements of the given matrix in the text fields and then click the button "Solution". If we want to see the check, we click the button "Check". Below we give the solution as it is presented by the computer program. We omit a part of the check.

**Example 1.** Find the inverse of matrix  $A$  using Gauss-Jordan elimination method, where

$$A = \begin{bmatrix} -1 & 3 & 1 \\ 2 & 1 & 4 \\ 1 & -2 & 0 \end{bmatrix}.$$

**Solution.** In the solution below  $a_{ij}$  represents the entry in a matrix located in the  $i$ -th row and the  $j$ -th column. First step: We form the block matrix:

$$\left[ \begin{array}{ccc|ccc} -1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array} \right]$$

Next step: To make  $a_{21} = 0$ , multiply the first row by 2 and add it to the second row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} -1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 7 & 6 & 2 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array} \right]$$

Next step: To make  $a_{31} = 0$ , add the first row to the third row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} -1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 7 & 6 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array} \right]$$

Next step: To make  $a_{11} = 1$ , multiply the first row by  $-1$ :

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & -3 & -1 & -1 & 0 & 0 \\ 0 & 7 & 6 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array} \right]$$

Next step: To make  $a_{22} = 1$ , we change the places of the second and the third row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & -3 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 7 & 6 & 2 & 1 & 0 \end{array} \right]$$

Next step: To make  $a_{32} = 0$ , multiply the second row by  $-a_{32} = -7$  and add it to the third row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & -3 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & -5 & 1 & -7 \end{array} \right]$$

Next step: To make  $a_{12} = 0$ , multiply the second row by  $-a_{12} = 3$  and add it to the first row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 0 & 3 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & -5 & 1 & -7 \end{array} \right]$$

Next step: To make  $a_{23} = 0$ , add the third row to the second row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 0 & 3 \\ 0 & 1 & 0 & -4 & 1 & -6 \\ 0 & 0 & -1 & -5 & 1 & -7 \end{array} \right]$$

Next step: To make  $a_{13} = 0$ , multiply the third row by 2 and add it to the first row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 2 & -11 \\ 0 & 1 & 0 & -4 & 1 & -6 \\ 0 & 0 & -1 & -5 & 1 & -7 \end{array} \right]$$

Next step: To make  $a_{33} = 1$ , multiply the third row by  $-1$ :

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 2 & -11 \\ 0 & 1 & 0 & -4 & 1 & -6 \\ 0 & 0 & 1 & 5 & -1 & 7 \end{array} \right]$$

Answer: The inverse matrix is

$$A^{-1} = \begin{bmatrix} -8 & 2 & -11 \\ -4 & 1 & -6 \\ 5 & -1 & 7 \end{bmatrix}.$$

Check. We will show that  $A.A^{-1} = I$ , where  $I$  is the identity matrix. Let  $c_{ij}$  represents the entry in the product  $A.A^{-1}$  located in the  $i$ -th row and the  $j$ -th column. Then

$$\begin{aligned} c_{11} &= (-1)(-8) + (3)(-4) + (1)(5) = 8 + (-12) + 5 = 1 \\ c_{12} &= (-1)(2) + (3)(1) + (1)(-1) = -2 + 3 + (-1) = 0 \\ c_{13} &= (-1)(-11) + (3)(-6) + (1)(7) = 11 + (-18) + 7 = 0 \end{aligned}$$

Hence

$$A.A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## 4. SECTIONS

The computer program contains four sections: "Matrices", "Examples", "Special Matrices" and "Random Matrices".

4.0.1. *Matrices.* In this section, we have to enter the elements of the matrix in the text fields. The text fields accept only integers or common fractions. Then we have to click either the button "Solution" or the button "Check".

The computer program has two algorithms for solving the inverse matrix of a given matrix. The first algorithm is partially illustrated in the above example 1. A complete description of the algorithm is give in the page "algorithm".

In the same page "algorithm" the student may find links to pages with the second algorithm. The second algorithm is more simple and straightforward. Below we illustrate the first operation in the second algorithm.

**Example 1a.** Let  $A$  be the matrix give in Example 1. Then the first operation on the augmented matrix gives

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & -3 & -1 & -1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array} \right]$$

Note that during the calculations we arrive at common fractions, in contrast to the first algorithm.

The first algorithm is closer to the work of the students during seminars. That is why the computer program is designed to use it.

If the given matrix does not have an inverse, the calculations end with a message that "the given matrix has no inverse matrix.". Below we give a simple example.

**Example 2.** Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 7 & 5 & 11 \end{bmatrix}.$$

**Solution.** In the solution below  $a_{ij}$  represents the entry in a matrix located in the  $i$ -th row and the  $j$ -th column. First step: We form the block matrix:

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 6 & 0 & 1 & 0 \\ 7 & 5 & 11 & 0 & 0 & 1 \end{array} \right]$$

Next step: To make  $a_{21} = 0$ , multiply the first row by  $-a_{21} = -2$  and add it to the second row:

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 7 & 5 & 11 & 0 & 0 & 1 \end{array} \right]$$

Since all entries in the second row of the left part of the block matrix are 0's, the left part of the block matrix cannot be reduced to the identity matrix. Hence, the given matrix has no inverse matrix.

Check. The given matrix has no inverse matrix. The check is given only if the matrix has an inverse matrix.  $\square$

4.0.2. *Examples.* In this section the user does not enter a matrix. He clicks on a corresponding button and the program shows the matrix and the solution.

The matrices from this section are from textbooks. In the computer program are implemented problems from [1, 2, 3].

4.0.3. *Special Matrices.* In this section the user does not enter a matrix. He clicks on a corresponding button and the program shows the matrix and the solution.

The matrices from this section are special matrices which are popular in the textbooks. These are the following subsections: “Vandermonde Matrix”, “Hilbert Matrix”, “Hankel Matrix”, “Fibonacci Matrix” and “Pascal Matrix”. We give here an example. Let’s click on the button “Pascal matrix of order 4”. We obtain the Pascal matrix of order 4 and the solution. Below we give as an example the Pascal matrix of order 4 and the answer:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 4 & -6 & 4 & -1 \\ -6 & 14 & -11 & 3 \\ 4 & -11 & 10 & -3 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

4.0.4. *Random Matrices.* In this section the user does not enter a matrix. He clicks on a corresponding button and the program shows the matrix and the solution.

The matrices from this section are random matrices, whose elements are produced accidentally from the computer program. Every new click on the button produces new random matrix and its solution and check. We could choose the elements of a random matrix from a given set of numbers. The available sets are as follows:  $\{0,1\}$ ,  $\{-1,1\}$ ,  $\{-1,0,1\}$ ,  $\{-9,\dots,0,\dots,9\}$ ,  $\mathbb{Q}$ .

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